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A Cooperative Game for the Realized Profit of an Aggregation of Renewable Energy Producers

Pratyush Chakraborty, Enrique Baeyens, Pramod P. Khargonekar, and Kameshwar Poolla

Abstract— **The aggregation of renewable energy has significant potential to mitigate undesirable characteristics such as intermittency and variability and thereby facilitate grid integration. Using cooperative game theory, it has been shown that aggregation is also beneficial for renewable energy producers because they can increase their expected profit by making a coalition, bidding a joint contract that maximizes the expected profit and sharing the profit in a way that keeps the game stable. However, we show that the realized (as opposed to expected) profit of the coalition, using the contract that maximizes the expected profit, cannot be suitably distributed among its members. We propose an alternative coalition contract and prove that it allows for a satisfactory distribution of the realized profit among the coalition members keeping the game stable. We design a new payoff allocation that lies in the core of the game of the realized profit. Finally, we analyze the cost of stabilizing the game by evaluating the loss of expected profit that a coalition incurs by bidding the stabilizing contract.**

I. Introduction

Sustainable social and economic development requires affordable access to the energy resources necessary to meet basic human needs and to serve productive processes without jeopardizing wellbeing of future generations. However, most of the current energy driving global economies comes from the combustion of fossil fuel that accounts for 56.6% of the greenhouse gas emissions [1]. Renewable energy resources, particularly wind and solar, offer a promising potential to reduce greenhouse gas emissions by displacing fossil fuel sources for electricity production. The main challenge for large-scale integration of wind and solar generation into the electric grid lies in their inherent variability, uncertainty, and nondispatchability [2]–[4].

The aggregation of geographically disperse sources offers a solution to the variability of renewable energy. Wind and solar power are usually negatively correlated in geographically near locations, while wind power tends to decorrelate as the distance increases [5]. Besides, aggregation provides economical benefit for the producers of renewable energy. The economic benefit of wind power aggregation was studied in a two-settlement market setting in [6] using cooperative game theory. The expected profit of a coalition can always be improved if a set of independent wind power producers decide to make a joint bid for a given time interval. It was also proved that there always exists a fair payoff allocation of the expected profit that is satisfactory for every coalition member. However, the expected profit is a theoretical quantity and the actual realized profit that is obtained in any given run can be very different from the expected profit. It is possible to devise payoff allocation mechanisms of the realized profit in such a way that the payment that each member receives approaches almost surely a suitable payoff allocation as the number of time intervals increases. Nevertheless, there are two reasons that can hinder the formation or continued operation of stable coalitions. The first reason is that the coalition members need to share and agree upon statistical models of their production. Second, the realized profit could turn out to be very different from the expected value during many time intervals over a long period. In such a case, one or more renewable energy producers may become dissatisfied with the profit payoff allocation mechanism and decide to abandon the coalition. To deal with these issues, some researchers have proposed a competitive game approach [7], [8].

In this paper, we continue to develop our previous cooperative game framework in the setting of realized profit. We propose a new joint contract that ensures long-term stable coalitions because it provides a superadditive value for the cooperative game of the realized profit. Consequently, the realized profit obtained by a coalition bidding the new stabilizing contract in the two-settlement market is greater than that obtained by the coalition members if they act independently trying to improve their realized profits. The cooperative game is also balanced. Besides, the producers do not need to agree on a common statistical framework. Each of them only uses statistical information about its own production and need not share it with the rest of the coalition members. In addition, we are able to design a payoff allocation for the game of the realized profit under the stabilizing contract that does not require to solve any linear program. However, the new stabilizing contract has a drawback. Since it does not maximize the expected profit of the coalition, there is a loss as compared to maximum expected profit. We are able to quantify this loss.

The remainder of this paper is organized as follows. A brief review of cooperative game theory, a summary of previous results about aggregation of renewable energy using cooperative games, its limitations and the formulation of

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Pratyush Chakraborty is with the Department of Electrical and Computer Engineering, University of Florida, Gainesville, FL, USA pchakraborty@ufl.edu

Enrique Baeyens is with Instituto de las Tecnologías Avanzadas de la Producción, Universidad de Valladolid, Valladolid, Spain enrbae@eis.uva.es

Pramod P. Khargonekar is with the Department of Electrical and Computer Engineering, University of Florida, Gainesville, FL, USA ppk@ufl.edu

Kameshwar Poolla is with the Department of Mechanical Engineering, University of California, Berkeley, CA, USA poolla@berkeley.edu

the problem are given in Section II. The new results about the cooperative game of the realized profit are given in Section III. The cost of stabilizing the game of the realized profit is analyzed in Section IV. The results are illustrated using an example in Section V. Finally, the conclusions are presented in Section VI.

II. Background and problem formulation

A. Background: Cooperative Game Theory

Game theory deals with rational behavior of economic agents in a mutually interactive setting [9]. Broadly speaking, there are two major categories of games: non-cooperative games and cooperative games. Cooperative games have been used extensively in diverse disciplines such as social science, economics, philosophy, psychology and communication networks [10], [11].

Let $\mathcal{N} := \{1, 2, \dots, N\}$ denote a finite collection of players.

Definition 1 (Coalition): A *coalition* is any subset $\mathcal{S} \subseteq \mathcal{N}$. The number of players in a coalition $\mathcal{S}$ is denoted by its cardinality, $|\mathcal{S}|$. The *set of all possible coalitions* is defined as the power set $2^{\mathcal{N}}$ of $\mathcal{N}$. The *grand coalition* is the set of all players, $\mathcal{N}$.

Definition 2 (Game and value): A cooperative game is defined by a pair $(\mathcal{N}, v)$ where $v : 2^{\mathcal{N}} \to \mathbb{R}$ is the *value function* that assigns a real value to each coalition $\mathcal{S} \subseteq \mathcal{N}$. Hence, the *value of coalition* $\mathcal{S}$ is given by $v(\mathcal{S})$.

Definition 3 (Superadditive game): A cooperative game $(\mathcal{N}, v)$ is *superadditive* if, for any pair of disjoint coalitions $\mathcal{S}, \mathcal{T} \subset \mathcal{N}$ with $\mathcal{S} \cap \mathcal{T} = \emptyset$, we have $v(\mathcal{S}) + v(\mathcal{T}) \leq v(\mathcal{S} \cup \mathcal{T})$.

If the value of the coalition $v(\mathcal{S})$ is *transferable*, the central question is how to *fairly* distribute it among the coalition members.

Definition 4 (Payoff allocation): A *payoff allocation* for the coalition $\mathcal{S} \subseteq \mathcal{N}$ is a vector $x \in \mathbb{R}^N$ whose entry x_i represents the payment to member $i \in \mathcal{S}$ ($x_i = 0, \quad i \notin \mathcal{S}$).

Definition 5 (Dissatisfaction and excess): The *dissatisfaction* of a coalition $\mathcal{S}$ with respect to the payoff allocation x is measured by the *excess* defined as follows:

$$e(x, \mathcal{S}) = v(\mathcal{S}) - \sum_{i \in \mathcal{S}} x_i. \tag{1}$$

An allocation is called *efficient* if its excess is zero.

Definition 6 (Imputation): A payoff allocation x for the grand coalition $\mathcal{N}$ is said to be an *imputation* if it is simultaneously efficient – *i.e.* $e(x, \mathcal{N}) = 0$, and individually rational – *i.e.* $e(x, \{i\}) \leq 0, \forall i \in \mathcal{N}$. Let $\mathcal{I}$ denote the set of all imputations.

The fundamental solution concept for cooperative games is the *core* that is analogous to the Nash equilibrium for non-cooperative games [9].

Definition 7 (The Core): The *core* $\mathcal{C}$ for the cooperative game $(\mathcal{N}, v)$ with *transferable payoff* is defined as the set of imputations such that no sub-coalition can obtain a payoff which is better than the sum of the members current payoffs under the given imputation.

$$\mathcal{C} := \left\{ x \in \mathcal{I} : e(x, \mathcal{N}) = 0, e(x, \mathcal{S}) \leq 0, \forall \mathcal{S} \in 2^{\mathcal{N}} \right\} \tag{2}$$

Games can have empty cores. Two important classes of games with nonempty cores are *convex games* and *balanced games* –the latter being a superset of the former.

Theorem 1 ([12]): A cooperative game has a *nonempty core* if it is *convex* –*i.e.*, has a *supermodular* value function,

$$v(\mathcal{S}) + v(\mathcal{T}) \ \leq \ v(\mathcal{S} \cup \mathcal{T}) + v(\mathcal{S} \cap \mathcal{T}), \quad \text{for all} \ \ \mathcal{S}, \mathcal{T} \subset \mathcal{N} \tag{3}$$

Convexity of a cooperative game is a strong condition and many real-world games are not convex. A weaker condition is balancedness of a cooperative game.

Definition 8 (Balanced game and balanced map): A game $(\mathcal{N}, v)$ is *balanced* if for any balanced map α, $\sum_{\mathcal{S} \in 2^{\mathcal{N}}} \alpha(\mathcal{S}) v(\mathcal{S}) \leq v(\mathcal{N})$ where the map $\alpha : 2^{\mathcal{N}} \to [0, 1]$ is said to be *balanced* if for all $i \in \mathcal{N}$, we have $\sum_{\mathcal{S} \in 2^{\mathcal{N}}} \alpha(\mathcal{S}) \mathbf{1}\{i \in \mathcal{S}\} = 1$.

However, not every cooperative game is balanced. For such games, alternative solution concepts have been proposed. The most prominent being the *Shapley value* and the *nucleolus*. The *Shapley value* offers an axiomatic approach to define a solution to a cooperative game. It is the only imputation that satisfies a set of five axioms and is given by the following analytical expression:

$$\chi_i(v) = \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus \{i\}} \frac{|\mathcal{S}|!(N - |\mathcal{S}| - 1)!}{N!} \left[v(\mathcal{S} \cup \{i\}) - v(\mathcal{S}) \right] \tag{4}$$

The Shapley value of a convex game lies in the core. The *nucleolus* of a cooperative game $(\mathcal{N}, v)$ is the imputation that minimizes the *dissatisfaction* of the players using a lexicographic order. The nucleolus of a balanced game lies in the core.

B. The players: the renewable energy producers

Consider a set of N independent renewable energy producers (REP) indexed by $i \in \mathcal{N}$. The power generation of REP $i \in \mathcal{N}$ is modeled as a scalar valued stochastic process $w_i(t) \in [0, W_i]$ where W_i is the nameplate production capacity of REP i. Each REP aims to maximize its profit by selling its renewable energy. A coalition is any set of players and the grand coalition is the set of all players $\mathcal{N}$. The power produced by a coalition $\mathcal{S}$ of renewable energy producers at time $t \in \mathbb{R}$ is $w_{\mathcal{S}}(t) = \sum_{i \in \mathcal{S}} w_i(t)$ with support $[0, W_{\mathcal{S}}]$, where $W_{\mathcal{S}} = \sum_{i \in \mathcal{S}} W_i$, and known cumulative distribution function denoted by $\Phi_{\mathcal{S}}(x, t) = P(w_{\mathcal{S}}(t) \leq x)$.

C. The market model

The set of REPs participates in a competitive two-settlement market system consisting of an ex-ante market consisting of the day-ahead (DA) forward market and an ex-post imbalance mechanism to penalize uninstructed contract deviations. The clearing price in the DA forward market is denoted by $p \in \mathbb{R}_+$. Deviations from the contract $C_{\mathcal{S}}$ are penalized ex-post at two different prices $(q, \lambda) \in \mathbb{R}_+^2$ depending if the deviation is negative (shortfall) or positive (surplus), respectively. The renewable energy producers are assumed to behave as price takers and the time-average imbalance prices in the interval $[t_0, t_f]$ are modeled as statistically independent nonnegative random variables $(q, \lambda) \in \mathbb{R}^2_+$ of the power production $w_{\mathcal{S}}$ of the coalition. This is a reasonable assumption in an scenario of low penetration of renewable energy. In markets with large penetration of renewable energy, the correlation of the prices with the energy production of the coalition increases with the size of the coalition.

D. Maximum expected profit

Let $\Pi(w_{\mathcal{S}}, C_{\mathcal{S}}, p, q, \lambda, t_0, t_f)$ denote the profit obtained by a coalition $\mathcal{S}$ for an offered joint contract $C_{\mathcal{S}}$ on the interval $[t_0, t_f]$ of length $T = t_f - t_0$ when the realized renewable power is $w_{\mathcal{S}} = \{w_{\mathcal{S}}(t) : t \in [t_0, t_f]\}$, the clearance market price is p and the prices for negative and positive imbalance are q and λ, respectively. In the sequel and for notational ease, we will only show the dependence of the coalition profit on the variables $w_{\mathcal{S}}$ and $C_{\mathcal{S}}$. The profit of a coalition $\mathcal{S}$ satisfies the following equation

$$\Pi(w_{\mathcal{S}}, C_{\mathcal{S}}) = \int_{t_0}^{t_f} \pi_t(w_{\mathcal{S}}(t), C_{\mathcal{S}})dt \qquad (5)$$

where

$$\begin{aligned}&\pi_t(w_{\mathcal{S}}(t), C_{\mathcal{S}})\\ &\quad = pC_{\mathcal{S}} - q(C_{\mathcal{S}} - w_{\mathcal{S}}(t))_+ - \lambda(w_{\mathcal{S}}(t) - C_{\mathcal{S}})_+\end{aligned} \qquad (6)$$

and $(x)_+ := \max\{0, x\}$. Equation (6) establishes that the profit equals the income obtained by selling the joint contract of power $C_{\mathcal{S}}$ minus the penalty for uninstructed deviation.

The profit is a random variable and its expected value $\mathbb{E}[\Pi(w_{\mathcal{S}}, C_{\mathcal{S}})]$ can be maximized by choosing the appropriate contract $C_{\mathcal{S}}$. The following lemma provides an explicit expression of the maximizing contract and the maximum expected profit for the important case of $p \le \mathbb{E}[q]$.

Lemma 1 ([6], [13]): The contract that maximizes the expected profit of a coalition $\mathcal{S}$ is given by the quantile function

$$C^*_{\mathcal{S}} = F^{-1}_{\mathcal{S}}(\gamma) = \inf\{x \in [0, W_s] : \gamma \le F_{\mathcal{S}}(x)\} \qquad (7)$$

where

$$\gamma = \frac{p + \mathbb{E}[\lambda]}{\mathbb{E}[q] + \mathbb{E}[\lambda]} \qquad (8)$$

and

$$F_{\mathcal{S}}(x) = \frac{1}{T}\int_{t_0}^{t_f} \Phi(x, t)dt \qquad (9)$$

The maximum expected profit is given by

$$\begin{aligned}&\frac{1}{T}\mathbb{E}[\Pi^*(w_{\mathcal{S}})]\\ &\quad = \frac{1}{T}\mathbb{E}[\Pi(w_{\mathcal{S}}, C^*_{\mathcal{S}})]\\ &\quad = \mathbb{E}[q]\int_0^{\gamma} F^{-1}_{\mathcal{S}}(x)dx - \mathbb{E}[\lambda]\int_{\gamma}^{1} F^{-1}_{\mathcal{S}}(x)dx\end{aligned} \qquad (10)$$

E. A cooperative game of renewable energy producers

A cooperative game of renewable energy producers can be defined by the pair $(\mathcal{N}, v)$, where $\mathcal{N}$ is the set of players and $v : 2^{\mathcal{N}} \to \mathbb{R}_+$ is the value function that is given by the maximum expected profit $v(\mathcal{S}) = \mathbb{E}[\Pi(w_{\mathcal{S}}, C^*_{\mathcal{S}})]$. This cooperative game is superadditive and balanced [6], [14]. Superadditivity means that the larger the coalition size, the greater the profit of its members. Balancedness is equivalent to the existence of an imputation in the core of the game. Thus, the renewable energy producers can increase their individual payoffs if they form a coalition. In order to implement this cooperative strategy the energy producers need to agree upon the probability distribution of their joint production and distribute the expected profit according to a payoff allocation rule in the core of the game.

F. Realized profit

The maximum expected profit of a coalition is the average profit that is obtained for a large number of repeating bids under the same conditions and using the maximizing contract (7). It is therefore, a theoretical quantity because actually only one bid is made and only one realization of the power production occurs. Consequently, the realized profit can be far away from the maximum expected profit, but it is this realized profit (which could even be negative) which has to be allocated in a suitable way to every coalition member. A first question arises: Can the realized profit obtained using the joint maximizing contract $C^*_{\mathcal{S}}$ be allocated in such a way that no member has an incentive to break up the coalition? We shall show in Section III that the answer to this question is negative. Consequently, the contract maximizing the expected profit does not stabilize the game when its value function is the realized profit. Then, a second question arises. Does there exist some contract $C_{\mathcal{S}}$ such that the cooperative game for the realized profit has a nonempty core? A contract satisfying this property is called a stabilizing contract for the realized profit cooperative game.

Problem formulation: Given a set of renewable energy producers, find a stabilizing contract for the cooperative game of the realized profit.

Since the stabilizing contract does not maximizes the expected profit, a third question arises: what is the cost of stabilizing this game?

III. A COOPERATIVE GAME FOR THE REALIZED PROFIT

Consider the cooperative game of renewable energy producers $(\mathcal{N}, v)$ where the value function is the realized profit obtained using a given contract $C_{\mathcal{S}}$, *i.e.* $v(\mathcal{S}) = \Pi(w_{\mathcal{S}}, C_{\mathcal{S}})$ for any $\mathcal{S} \in 2^{\mathcal{N}}$. Let us begin by defining the concept of stabilizing contract for this game.

Definition 9 (Stabilizing contract): The contract $C_{\mathcal{S}}$ for any coalition $\mathcal{S} \in 2^{\mathcal{N}}$ is said to be *stabilizing* for the cooperative game $(\mathcal{N}, v)$ where the value function is the realized profit, *i.e.* $v(\mathcal{S}) = \Pi(w_{\mathcal{S}}, C_{\mathcal{S}})$, if the game has a nonempty core.

A. Instability of the contract that maximizes the expected profit

The contract $C_{\mathcal{S}}^*$ that maximizes the expected profit is not stabilizing for the cooperative game of the realized profit. We show it by devising a simple counterexample for a cooperative game of three REPs.

Counterexample 1 (Instability of the contract): Consider a group of three REPs. Each of them has a power production capacity normalized to one. Let us assume that $w_i(t),\ i \in \{1,2,3\}$ are independent uniform random variables with support in the unit interval $[0,1]$. For a given time interval of unit length $T = 1$, the market clearing price is $p = 1$ and the expected penalty prices are $\mu_q = \mathbb{E}[q] = 6$ and $\mu_\lambda = \mathbb{E}[\lambda] = 0$. The probability distribution of the joint power production of a coalition $\mathcal{S}$ of cardinality $|\mathcal{S}|$ is given by the Irwin-Hall distribution, with cumulative distribution function given by:

$$F_{\mathcal{S}}(w_{\mathcal{S}}(t)) = \frac{1}{|\mathcal{S}|!}\sum_{k=0}^{|\mathcal{S}|}(-1)^k\binom{|\mathcal{S}|}{k}(w_{\mathcal{S}}(t)-k)_+^{|\mathcal{S}|} \quad (11)$$

The contract size that maximizes the expected profit for each possible coalition can be computed by applying Lemma 1 for $\gamma = (p+\mu_\lambda)/(q+\mu_\lambda) = 1/6$. Since the producers have identical independent probability distributions for their power productions, all the coalitions of the same cardinality have the same maximizing contract.

TABLE I

EVERY POSSIBLE COALITION $\mathcal{S}_k$ WITH CONTRACTS C_k^* MAXIMIZING EXPECTED PROFIT, REALIZED POWER $w_k(t)$ AND REALIZED PROFIT π_t^k AT A TIME t FOR EACH POSSIBLE COALITION

k	$\mathcal{S}_k$	C_k^*	$w_k(t)$	π_t^k
1	$\{1\}$	$\frac{1}{6}$	0	0
2	$\{2\}$	$\frac{1}{6}$	0	0
3	$\{3\}$	$\frac{1}{6}$	1	$-\frac{2}{3}$
4	$\{1,2\}$	$\frac{1}{\sqrt{3}}$	0	0
5	$\{1,3\}$	$\frac{1}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}-1$
6	$\{2,3\}$	$\frac{1}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}-1$
7	$\{1,2,3\}$	1	1	0

Suppose that the realized produced powers at a certain time instant are $w_1(t) = 0$, $w_2(t) = 0$ and $w_3(t) = 1$ and the realized penalty prices are $q = \lambda = 1$. Then the realized joint power and the realized joint profit for each coalition are given in Table I. The cooperative game for the realized profit obtained using the contract maximizing the expected profit is not superadditive because

$$\pi_t^{\{1,2,3\}} < \pi_t^{\{1\}} + \pi_t^{\{2,3\}}$$

This counterexample shows that the contract that maximizes the expected profit is not stabilizing for the realized profit. Consequently, if several producers are dissatisfied with their allocated profit during a long periods of time, they can decide to leave the coalition. In addition, coalition formation under the contract that maximizes the expected profit requires that the renewable energy producers agree upon a common statistical model of the joint production. This can be also a limitation for coalition formation and power aggregation.

B. A contract that stabilizes the cooperative game for the realized profit

The contract that maximizes the expected profit of a coalition of renewable energy producers is not a stabilizing contract for the cooperative game of the realized profit. Here, we propose an alternative contract for the value function of the cooperative game. The new contract has several nice properties. First, the realized profit for this contract is a superadditive function. Second, the core of the cooperative game of the realized profit obtained using this contract is nonempty; in other words, the contract stabilizes the cooperative game for the realized profit. And third, the players of the game do not need to agree on a common distribution function for their joint power production. Thus, the new contract promotes the aggregation of renewable energy. But, the new contract is only suboptimal because it is not maximizing the expected profit of a coalition and hence, there is a loss of profit for the coalition members in the long term.

Let $\mathcal{S} \in 2^{\mathcal{N}}$ be any coalition of renewable energy producers, define the contract $C_{\mathcal{S}}^+$ as follows:

$$C_{\mathcal{S}}^+ = \sum_{i\in\mathcal{S}} C_i^* \quad (12)$$

where C_i^* denotes the maximizing contract for the renewable energy producer $i \in \mathcal{N}$ and is given by

$$C_i^* = F_i^{-1}(\gamma) \quad (13)$$

for the value of γ given by equation (8). Note that F_i denotes the CDF of the power production of producer i and F_i^{-1} is the quantile function given by equation (7) for $\mathcal{S} = \{i\}$.

The properties of the proposed game are inherited from the properties of the realized profit function $\Pi(w_{\mathcal{S}}, C_{\mathcal{S}})$ given by equation (6). These properties are summarized in the following lemma.

Lemma 2: The realized profit function $\Pi(w_{\mathcal{S}}, C_{\mathcal{S}})$ defined by equation (5) is positive homogeneous and superadditive.

Proof: To prove positive homogeneity, let $\alpha > 0$ be an arbitrary nonnegative scalar, and take into account that $(\cdot)_+$ is positive homogeneous, then

$$\begin{aligned}
&\pi_t(\alpha w_{\mathcal{S}}(t), \alpha C_{\mathcal{S}}) = \\
&\quad = p(\alpha C_{\mathcal{S}}) - q(\alpha C_{\mathcal{S}} - \alpha w_{\mathcal{S}}(t))_+ - \lambda(\alpha w_{\mathcal{S}}(t) - \alpha C_{\mathcal{S}}(t))_+ \\
&\quad = \alpha\left(pC_{\mathcal{S}} - q(C_{\mathcal{S}} - w_{\mathcal{S}}(t))_+ - \lambda(w_{\mathcal{S}}(t) - C_{\mathcal{S}})_+\right) \\
&\quad = \alpha\pi_t(w_{\mathcal{S}}(t), C_{\mathcal{S}})
\end{aligned}$$

by integrating both sides in the time interval $[t_0, t_f]$ we obtain the desired result.

To prove superadditivity, suppose we have arbitrary $w_{\mathcal{S}_1}, w_{\mathcal{S}_2}, C_{\mathcal{S}_1}, C_{\mathcal{S}_2} \in \mathbb{R}$. Let $w_{\mathcal{S}} = w_{\mathcal{S}_1} + w_{\mathcal{S}_2}$ and $C_{\mathcal{S}} = C_{\mathcal{S}_1} + C_{\mathcal{S}_2}$. Now, taking into account that q and λ are nonnegative and $(\cdot)_+$ is a subadditive function

$$\begin{aligned}
&\pi_t(w_{\mathcal{S}}(t), C_{\mathcal{S}}) \\
&\quad = pC_{\mathcal{S}} - q(C_{\mathcal{S}} - w_{\mathcal{S}}(t))_+ - \lambda(w_{\mathcal{S}}(t) - C_{\mathcal{S}})_+ \\
&\quad \geq p(C_{\mathcal{S}_1} + C_{\mathcal{S}_2}) \\
&\qquad - q\left((C_{\mathcal{S}_1} - w_{\mathcal{S}_1}(t))_+ + (C_{\mathcal{S}_2} - w_{\mathcal{S}_2}(t))_+\right) \\
&\qquad - \lambda\left((w_{\mathcal{S}_1}(t) - C_{\mathcal{S}_1})_+ + (w_{\mathcal{S}_2}(t) - C_{\mathcal{S}_2})_+\right) \\
&\quad = pC_{\mathcal{S}_1} - q(C_{\mathcal{S}_1} - w_{\mathcal{S}_1}(t))_+ - \lambda(w_{\mathcal{S}_1} - C_{\mathcal{S}_1}(t))_+ \\
&\qquad + pC_{\mathcal{S}_2} - q(C_{\mathcal{S}_2} - w_{\mathcal{S}_2}(t))_+ - \lambda(w_{\mathcal{S}_2} - C_{\mathcal{S}_2}(t))_+ \\
&\quad = \pi_t(w_{\mathcal{S}_1}(t), C_{\mathcal{S}_1}) + \pi_t(w_{\mathcal{S}_2}(t), C_{\mathcal{S}_2})
\end{aligned}$$

Finally, we obtain the desired result by integrating in the time interval $[t_0, t_f]$. □

We propose now a cooperative game for the realized profit. Consider the cooperative game

$$G = (\mathcal{N}, v) \tag{14}$$

where the value function $v : 2^{\mathcal{N}} \to \mathbb{R}$ is defined as the realized profit for the contract $C_{\mathcal{S}}^+$ given by equation (12)

$$v(\mathcal{S}) = \Pi\left(w_{\mathcal{S}}, C_{\mathcal{S}}^+\right). \tag{15}$$

Our first result is that the cooperative game $G = (\mathcal{N}, v)$ is supperadditive. This is a consequence of the supperadditivity of the realized profit function. The main implication is that a coalition of independent REPs can improve its realized profit by making a joint bid in the two-settlement market.

Theorem 2: The cooperative game G is superadditive.

Proof: It is a consequence of the superadditivity of the realized profit proved in Lemma 2 □

As a result of the previous theorem, we have the following corollary.

Corollary 1: The realized profit of a coalition is always greater than or equal to the sum of realized profits of each members.

The producers do not need to agree upon a common statistical model to calculate a joint contract. Each of them can calculate the contract that maximizes its own expected profit independent of the other producers and they only agree in making a joint bid for the sum of the independent contracts. The resulting contract is not the best one, because it does not maximize the total expected profit of the coalition, however it allows to obtain a profit that is never less that the sum of the realized profits that each producer would get by bidding independently.

Our second result establishes that there always exist suitable pay allocation mechanisms to distribute the realized profit among the coalition members in such a way that no member has a reason to break up the coalition, *i.e.* the core of the cooperative is nonempty.

Theorem 3: The cooperative game $G = (\mathcal{N}, v)$ where v is the value function defined by equation (15) is balanced and hence has a nonempty core.

Proof: The game $(\mathcal{N}, v)$ is balanced if for any balanced map α, $\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})v(\mathcal{S}) \leq v(\mathcal{N})$. By using positive homogeneity of the realized profit function,

$$\begin{aligned}
\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})v(\mathcal{S}) &= \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})\Pi(w_{\mathcal{S}}, C_{\mathcal{S}}^+) \\
&= \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \Pi(\alpha(\mathcal{S})w_{\mathcal{S}}, \alpha(\mathcal{S})C_{\mathcal{S}}^+)
\end{aligned}$$

By using superaditivity of the realized profit function and taking into account that $C_{\mathcal{S}}^+ = \sum_{i\in\mathcal{S}} C_i^*$,

$$\begin{aligned}
&\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})v(\mathcal{S}) \\
&\quad \leq \Pi\left(\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})w_{\mathcal{S}}, \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})C_{\mathcal{S}}^+\right) \\
&\quad = \Pi\left(\sum_{i\in\mathcal{N}} \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})\mathbf{1}\{i\in\mathcal{S}\}w_i,\right. \\
&\qquad\qquad \left.\sum_{i\in\mathcal{N}} \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})\mathbf{1}\{i\in\mathcal{S}\}C_i^*\right) \\
&\quad = \Pi\left(\sum_{i\in\mathcal{N}} w_i, \sum_{i\in\mathcal{N}} C_i^*\right) \\
&\quad = \Pi(w_{\mathcal{N}}, C_{\mathcal{N}}^+) = v(\mathcal{N})
\end{aligned}$$

and this proves balancedness of the game. □

C. Nonconvexity of the cooperative game for the realized profit

We are interested in obtaining a payoff allocation of the realized profit that is in the core of the game. It is well-known that if a cooperative game is convex, then the Shapley value is an imputation that lies in the core. Unfortunately, the cooperative game of renewable energy producers for the realized profit obtained using the contract $C_{\mathcal{S}}^+$ is not convex. This can be easily proved by devising a counterexample where the imputation given by the Shapley value is not in the core.

Counterexample 2 (Convexity): Consider a cooperative game of three renewable energy producers with the same production capacity. Let us assume that each producer has complete knowledge of his own probability distribution, so he can calculate the contract $C_i^*(i \in \{1,2,3\})$ that maximize his expected profit for a certain time interval $[t_0, t_f]$. Let us assume for simplicity that the individual power productions are independent and identically distributed random variables and such that the individual contracts maximizing the expected profit are $C_1^* = C_2^* = C_3^* = 1$. Assume that the realized prices are $p = 1$, $q = 2p$ and $\lambda = 0$ and the realized productions are constant during the interval $[t_0, t_f]$ and given by $w_1 = w_2 = 0.4$ and $w_3 = 1.6$. The allocation given by the Shapley value is $x_1 = 0.5$, $x_2 = 0.5$, and $x_3 = 1.4$. In Table II, we show the realized profit of each possible coalition $v(\mathcal{S})$ and the excess under the Shapley value imputation that is given by $e = v(\mathcal{S}) - \sum_{i\in\mathcal{S}} x_i$. Since the excess of coalitions $\{1,3\}$ and $\{2,3\}$ are positive, the imputation given by the Shapley value is not in the core of the game. Consequently, the game is not convex.

TABLE II
SHAPLEY VALUE IMPUTATION

$\mathcal{S}$	$v(\mathcal{S})$	$e = v(\mathcal{S}) - \sum_{i\in\mathcal{S}} x_i$
$\{1\},\{2\}$	0.4	−0.1
$\{3\}$	1.0	−0.4
$\{1,2\}$	0.8	−1.0
$\{1,3\},\{2,3\}$	2.0	0.1
$\{1,2,3\}$	2.4	0.0

D. Sharing the realized profit: An imputation in the core

Since the cooperative game of the realized profit for the contract $C_{\mathcal{S}}^{+}$ is balanced, any imputation in the core provides a suitable payoff allocation for distributing the profit of a coalition. A payoff allocation that always lies in the core of a balanced game is the nucleolus. Unfortunately, its computation is very demanding because the lexicographic ordering minimization requires solving a sequence of $O(2^N)$ linear programs [15]. Another alternative is the payoff allocation that minimizes the worst-case excess that we previously introduced in [14], which only requires to solve one LP. However, we shall show that for the problem of sharing the realized profit, there is a remarkable imputation that lies in the core of the game and can be directly computed without solving any linear program.

Theorem 4: The payoff allocation of the realized profit given by

$$x_i = \int_{t_0}^{t_f} \xi_i(t)dt,\ i \in \mathcal{N}, \tag{16}$$

where

$$\xi_i(t) = \begin{cases} 0, & \text{if } D_{\mathcal{N}}(t) = 0 \\ pC_i^* - q\frac{C_i^* - w_i(t)}{|D_{\mathcal{N}}(t)|}(D_{\mathcal{N}}(t))_+ + & \\ \lambda\frac{C_i^* - w_i(t)}{|D_{\mathcal{N}}(t)|}(-D_{\mathcal{N}}(t))_+, & \text{if } D_{\mathcal{N}}(t) \neq 0 \end{cases}$$

and $D_{\mathcal{N}}(t) = \sum_{i\in\mathcal{N}}(C_i^* - w_i(t))$ lies in the core.

Proof: The core is the set of all imputations satisfying (2) where $e(x,\mathcal{S}) = \int_{t_0}^{t_f} \epsilon(\xi(t),\mathcal{S})dt$ and $\epsilon(\xi(t),\mathcal{S}) = \pi_t(w_{\mathcal{S}}(t), C_{\mathcal{S}}^{+}) - \sum_{i\in\mathcal{S}}\xi_i(t)$. Let $D_{\mathcal{S}}(t) = \sum_{i\in\mathcal{S}}(C_i^* - w_i(t))$. If $D_{\mathcal{N}}(t) = 0$, then $\epsilon(\xi(t),\mathcal{S}) = 0$ for all $\mathcal{S} \subseteq \mathcal{N}$, but if $D_{\mathcal{N}}(t) \neq 0$, then

$$\pi_t(w_{\mathcal{S}}(t), C_{\mathcal{S}}^{+}) = \sum_{i\in\mathcal{S}} pC_i^* - q(D_{\mathcal{S}}(t))_+ - \lambda(-D_{\mathcal{S}}(t))_+$$

$$\sum_{i\in\mathcal{S}}\xi_i(t) = \sum_{i\in\mathcal{S}} pC_i^* - q\frac{D_{\mathcal{S}}(t)}{|D_{\mathcal{N}}(t)|}(D_{\mathcal{N}}(t))_+ + \lambda\frac{D_{\mathcal{S}}(t)}{|D_{\mathcal{N}}(t)|}(-D_{\mathcal{N}}(t))_+$$

$$\epsilon(\xi(t),\mathcal{S}) = -q\left((D_{\mathcal{S}}(t))_+ - \frac{D_{\mathcal{S}}(t)}{|D_{\mathcal{N}}(t)|}(D_{\mathcal{N}}(t))_+\right) - \lambda\left((-D_{\mathcal{S}}(t))_+ + \frac{D_{\mathcal{S}}(t)}{|D_{\mathcal{N}}(t)|}(-D_{\mathcal{N}}(t))_+\right)$$

For $\mathcal{S} \neq \mathcal{N}$, we can distinguish two cases: (i) if $D_{\mathcal{S}}(t)/D_{\mathcal{N}}(t) > 0$ then $\epsilon(\xi(t),\mathcal{S}) = 0$, (ii) if $D_{\mathcal{S}}(t)/D_{\mathcal{N}}(t) \leq 0$, then $\epsilon(\xi(t),\mathcal{S}) = -(q+\lambda)|D_{\mathcal{S}}(t)| \leq 0$. For $\mathcal{S} = \mathcal{N}$, always $D_{\mathcal{S}}(t)/D_{\mathcal{N}}(t) = 1 > 0$ and $\epsilon(\xi(t),\mathcal{N}) = 0$. By integrating in the interval $[t_0, t_f]$, $e(x(t),\mathcal{S}) \leq 0$ for all $\mathcal{S} \subset \mathcal{N}$ and $e(x(t),\mathcal{N}) = 0$ and this proves that the allocation lies in the core. □

IV. Cost of Stabilizing the Game of the Realized Profit

In this section, we study the cost of stabilizing the cooperative game of REPs for the realized profit. Before the interval $[t_0, t_f]$ occurs, the renewable power production of a coalition $\mathcal{S}$ is a random variable $w_{\mathcal{S}}$. Hence, the profit is also a random variable.

Definition 10 (Maximum expected benefit of a coalition): The maximum expected benefit of a coalition $\mathcal{S}$ is defined as the maximum increase of expected profit that the coalition can obtain by bidding a joint contract in the day ahead market and is denoted by $B_{\mathcal{S}}^*$.

The maximum expected benefit of a coalition $\mathcal{S}$ is the difference between the maximum expected profit obtained by the coalition and the maximum expected profit obtained by its members if they would act independently and is given by

$$B_{\mathcal{S}}^* = \mathbb{E}\left[\Pi^*(w_{\mathcal{S}})\right] - \sum_{i\in\mathcal{S}}\mathbb{E}\left[\Pi^*(w_i)\right] \tag{17}$$

By substitution of the expression of the maximum expected profit given in Lemma 1,

$$B_{\mathcal{S}}^* = T\mathbb{E}\left[q\right]\int_0^{\gamma}\left(F_{\mathcal{S}}^{-1}(\theta) - \sum_{i\in\mathcal{S}}F_i^{-1}(\theta)\right)d\theta - T\mathbb{E}\left[\lambda\right]\int_{\gamma}^{1}\left(F_{\mathcal{S}}^{-1}(\theta) - \sum_{i\in\mathcal{S}}F_i^{-1}(\theta)\right)d\theta \tag{18}$$

The maximum expected benefit is obtained when the coalition bids the contract that maximizes the expected profit $C_{\mathcal{S}}^*$. However, we showed in Section III that this contract does not stabilize the realized profit game. For any other joint contract $C_{\mathcal{S}}$ there is a loss as compared to the maximum expected profit of the coalition.

Definition 11 (Loss of expected profit of a coalition): The loss of expected profit of a coalition $\mathcal{S}$ bidding a contract $C_{\mathcal{S}}$ is denoted by $L_{\mathcal{S}}(C_{\mathcal{S}})$ and it is the difference between its expected profit and the maximum expected profit.

The loss of a coalition $\mathcal{S}$ that bids the contract $C_{\mathcal{S}}$ in the day ahead market is given by

$$L_{\mathcal{S}}(C_{\mathcal{S}}) = \mathbb{E}\left[\Pi^*(w_{\mathcal{S}}) - \Pi(w_{\mathcal{S}}, C_{\mathcal{S}})\right] \tag{19}$$

An analytical expression of its value is given in the following theorem.

Theorem 5: The loss of a coalition $\mathcal{S}$ by bidding a contract $C_{\mathcal{S}}$ is given by

$$L_{\mathcal{S}}(C_{\mathcal{S}}) = T\mathbb{E}\left[q+\lambda\right]\int_{F_{\mathcal{S}}(C_{\mathcal{S}})}^{\gamma}\left(F_{\mathcal{S}}^{-1}(\theta) - C_{\mathcal{S}}\right)d\theta \tag{20}$$

Proof: By doing the change of variables $\theta = F_{\mathcal{S}}(w)$, and after some manipulation the expression of the expected profit becomes

$$\begin{aligned}\frac{1}{T}\mathbb{E}\left[\Pi(w_{\mathcal{S}}, C_{\mathcal{S}})\right] &= \mathbb{E}\left[q\right]\int_0^{\gamma} F_{\mathcal{S}}^{-1}(\theta)d\theta - \mathbb{E}\left[\lambda\right]\int_{\gamma}^{1} F_{\mathcal{S}}^{-1}(\theta)d\theta \\ &\quad - \mathbb{E}\left[q+\lambda\right]\int_{F_{\mathcal{S}}C_{\mathcal{S}})}^{\gamma}\left(F_{\mathcal{S}}^{-1}(\theta) - C_{\mathcal{S}}\right)d\theta\end{aligned}$$

It is not difficult to prove using the inverse change of variables $w = F^{-1}(\theta)$ that

$$\int_{F_{\mathcal{S}}(C_{\mathcal{S}})}^{\gamma}\left(F_{\mathcal{S}}^{-1}(\theta) - C_{\mathcal{S}}\right)d\theta \geq 0 \tag{21}$$

The expected profit is expressed as the difference of two terms, where the first term does not depend on the decision variable $C_{\mathcal{S}}$ and the second one is always nonnegative. Consequently, the first term is the maximum expected profit that is obtained by choosing the contract that makes the second term zero, *i.e.* for $C_{\mathcal{S}} = C_{\mathcal{S}}^*$ such that $F_{\mathcal{S}}(C_{\mathcal{S}}^*) = \gamma$

$$\frac{1}{T}\mathbb{E}\left[\Pi^*(w_{\mathcal{S}})\right] = \mathbb{E}\left[q\right]\int_0^{\gamma} F_{\mathcal{S}}^{-1}(\theta)d\theta - \mathbb{E}\left[\lambda\right]\int_{\gamma}^{1} F_{\mathcal{S}}^{-1}(\theta)d\theta$$

and the loss of expected profit is given by equation (20). □

Remark 1: In addition to the expression of the loss of expected profit of a coalition, this theorem also provides an elegant proof for the closed-form expressions of the optimal expected profit and the corresponding maximizing contract. Note that optimality is provided by positivity of (21) instead of using first and second order optimality conditions, see Theorem IV.1 in [13].

In order to evaluate the cost of stabilizing the cooperative game of the realized profit by using the contract $C_{\mathcal{S}}^+$ given by equation (12), we shall obtain the loss of expected profit that a coalition $\mathcal{S}$ incurs by bidding this contract in the day ahead market. The result is obtained by direct substitution of $C_{\mathcal{S}}^+$ in equation (20) and is summarized in the following corollary.

Corollary 2: The loss of expected profit of a coalition $\mathcal{S}$ by bidding the stabilizing contract for the realized profit $C_{\mathcal{S}}^+$ defined by equation (12) is

$$L_{\mathcal{S}}^+ = T\mathbb{E}\left[q+\lambda\right]\int_{F_{\mathcal{S}}(C_{\mathcal{S}}^+)}^{\gamma}\left(F_{\mathcal{S}}^{-1}(\theta) - C_{\mathcal{S}}^+\right)d\theta \tag{22}$$

Stabilizing the cooperative game of the realized profit has a cost that is measured by the loss of expected benefit. Consequently, the cooperative game of the realized profit cannot attain the maximum expected benefit of aggregation. However, since the value function given by the realized profit is a superadditive function, there is still an expected benefit of aggregation. For a coalition $\mathcal{S} \subseteq \mathcal{N}$, the expected benefit obtained by bidding the stabilizing contract $C_{\mathcal{S}}^+$ is $B_{\mathcal{S}}^+ = B_{\mathcal{S}}(C_{\mathcal{S}}^+)$ and can be calculated as the difference between the maximum expected benefit of the coalition and the loss incurred by using the stabilizing contract $C_{\mathcal{S}}^+$, *i.e.*

$$B_{\mathcal{S}}^+ = B_{\mathcal{S}}^* - L_{\mathcal{S}}^+ \tag{23}$$

where $B_{\mathcal{S}}^*$ and $L_{\mathcal{S}}^+$ are given by equations (18) and (22), respectively.

V. An Example

In this example, we consider three wind power producers. We assume that they are geographically distant in order to consider that the probability distribution of their wind power productions are independent. We also assume that they have the same nameplate capacity and the wind power production (normalized by the nameplate capacity) is modeled by a Gamma random variable with parameters $a = 5$ and $b = 1/15$, *i.e.* $w_i \sim \Gamma(5, \frac{1}{15})$. The gamma distribution has been frequently used in the literature to model wind speed [16]–[18].

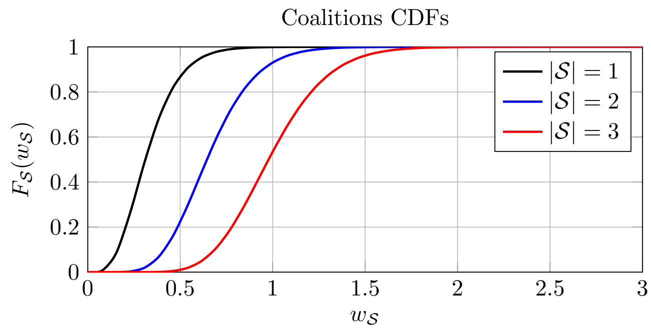

Fig. 1. CDFs of the different possible coalitions

For simplicity, the power production random variables are assumed to be constant in the time interval $[t_0, t_f]$. The cumulative probability distributions for every possible coalition are depicted in Figure 1. Since the individual power productions are independent and identically distributed random variables, the joint power production of coalitions of the same cardinality are also independent and identically distributed random variables.

Let us assume that the clearing market price is $p = 1$, the penalty price for shortfall is $q = 3$ and the penalty price for excess is $\lambda = 0$, which is the case if the producers have curtailment capability.

TABLE III
THE BENEFIT OF AGGREGATION

$\lvert\mathcal{S}\rvert$	$C_{\mathcal{S}}^*$	$C_{\mathcal{S}}^+$	$B_{\mathcal{S}}^*$	$L_{\mathcal{S}}^+$	$B_{\mathcal{S}}^+$
1	0.2537	0.2537	0	0	0
2	0.5596	0.5075	0.0795	0.0517	0.0278
3	0.8722	0.7612	0.1753	0.1271	0.0481

A summary of the results for each possible coalition in this example is shown in Table III. In this table, we show the values of the contract that maximizes the expected profit $C_{\mathcal{S}}^*$ and the stabilizing contract $C_{\mathcal{S}}^+$ for the realized profit, the maximum expected increase of profit $B_{\mathcal{S}}^*$, the loss of expected profit $L_{\mathcal{S}}^+$ for the stabilizing contract and the expected increase of profit $B_{\mathcal{S}}^+$ for the stabilizing contract.

TABLE IV
REALIZED PROFIT FOR TEN REALIZATIONS

#	$\Pi(w_1)$	$\Pi(w_2)$	$\Pi(w_3)$	$\Pi(w_{\mathcal{N}})$
1	0.2537	0.2537	0.2537	0.7612
2	0.2537	−0.2930	0.2537	0.7612
3	0.1465	0.0675	0.2537	0.6658
4	0.0617	0.2537	0.2537	0.7612
5	−0.2531	−0.2002	0.2537	0.6604
6	0.2537	0.2537	−0.0985	0.7612
7	−0.1602	0.0709	0.2537	0.2377
8	−0.1322	0.2537	0.1816	0.3535
9	−0.1909	0.2537	0.2537	0.7612
10	−0.1550	0.2537	−0.0238	0.4692

TABLE V
REALIZED PROFIT ALLOCATION FOR TEN REALIZATIONS

#	x_1	x_2	x_3	$e(x,\mathcal{N})$
1	0.2537	0.2537	0.2537	0
2	0.2537	0.2537	0.2537	0
3	0.1465	0.0675	0.4519	0
4	0.2537	0.2537	0.2537	0
5	−0.2531	−0.2002	1.1137	0
6	0.2537	0.2537	0.2537	0
7	−0.1602	0.0709	0.3270	0
8	−0.1322	0.3041	0.1816	0
9	0.2537	0.2537	0.2537	0
10	−0.1550	0.6480	−0.0238	0

Finally in Tables IV and V, we show the realized power and the corresponding payoff allocations for the grand coalition of the cooperative game of the realized profit and for ten different wind power realizations. The realized profits for each independent wind power producer and for the grand coalition is given in Table IV. The profit allocation for each wind power realization using the payoff allocation of Theorem 4. provide an imputation in the core is given in Table V. Note that the excess e is always nonnegative, and this ensures that the corresponding imputation belongs to the core of the game.

VI. CONCLUSIONS

Aggregation of renewable energy resources allows to alleviate some of its limitations for integration in the grid. Besides, it also provides economic benefit by increasing the expected profit that a coalition of renewable energy producers obtain in a two-settlement market. However, the realized profit can be very different from the expected profit on a given time interval and may not be allocated in a satisfactory way for every coalition member. Moreover, the coalition members need to agree upon a common statistical framework to design the optimal contract. Both reasons can contribute to destabilize a coalition. To avoid these issues, a stabilizing contract and a new cooperative game for the realized profit has been proposed. Assuming transferable payoff and a function value defined as the realized profit obtained by the proposed stabilizing contract, we proved that the corresponding cooperative game is superadditive and balanced. Thus, there exist suitable payoff allocations of the realized profit at every time interval that are satisfactory for all the coalition members. We design a computationally efficient stabilizing payoff allocation that does not require to solve any linear program. In addition, we were able to quantify the loss of expected profit incurred by using the proposed strategy. The loss of expected profit is the price that the coalition has to pay for ensuring stability and profitability at every time interval.

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