A Demand Response Game and its Robust Price of Anarchy*

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***Abstract*—Increased variability in power generation due to large scale integration of renewable energy sources such as wind and solar power is a significant technical challenge in power systems operations and control. In addition, there is a compelling value in reducing the peak demand since it occurs only for a small fraction of time, while the power system is designed to reliably satisfy the peak demand. One promising approach to reduce variability of renewable generation and peak demand is to harness the *inherent flexibility* of electric power loads of consumers. Efficient control techniques are required to manage flexibility in consumer demands. Advancements in sensing, communications and computational technologies infused into the power system resulting in the *cyber-physical-social electric grid,* are creating opportunities for novel control solutions. In this paper, we first formulate a centralized demand side management approach. Next, we consider a decentralized approach for controlling the loads where the flexible load consumers play a non-cooperative game among each other. We show that Nash equilibria exist for this game. Our main technical result is that the demand response game in decentralized approach has the property of being a *valid monotone utility game.* This in turn leads to robust lower bounds on the price of anarchy (POA) for our game.**

I. INTRODUCTION

Climate change, green house gas emissions and sustainability along with rapidly decreasing costs are some of the main motivating reasons for introducing renewable energy sources such as wind and solar into the existing power system. But there are major hurdles that need to be overcome in order to integrate them and obtain the same reliability as that of the existing grid. The power outputs from these sources are uncertain (stochastic in nature), intermittent (have large fluctuations and ramps) and non-dispachable (cannot be controlled to follow a command). These three properties together are captured by the term *variability* [1]. The variability of renewables introduces major challenge in supply-demand balancing in case of their deep penetration in the grid [2].

The demand for electric power is increasing (rapidly in developing nations and more slowly in the developed nations) due to economic growth, population growth and introduction of new loads such as computers, electric vehicles and plug-in-hybrid-electric vehicles. As a result, peak power demand is also increasing, which is another pressing issue in the present grid. Though peak demand occurs for only for a small fraction of time of the year [3], since the power system must be designed to reliably satisfy the peak demand, increase in peak demand increases the overall cost of power system infrastructure.

A promising solution to reduce the variability of renewables and peak demand is obtained by controlling the consumers' demand. A paradigm shift in the power system operation is in progress where consumer loads will be actively participating in deciding supply schedule of generators. Harnessing the flexibility of the loads such as electric vehicles (EV), water heaters, washers, dryers, refrigerators, air-conditioning, etc., consumers will shape their demand following some signal from a control authority. This method of demand control is called *demand side management* (DSM) or *demand response* (DR) where advance sensing, computation and communication technologies have a big role to play. Potential of different electric loads such as electric vehicles, air conditioning etc. in electrical power demand management has been studied in detail in [4], [5], [6]. There are different system level formulations of achieving demand side management. Some of the important ones are [7], [8], [9], [10] etc. Exploiting flexibility of EVs and other flexible loads and imposing suitable prices, the total cost of energy consumption can be reduced as described in [8] and [10]. But these papers do not consider utility functions of the loads and consider that the total energy consumption of the flexible loads are fixed across the time duration. In [7], a distributed policy for demand response is introduced which aligns individual consumer's objective with social objective. But here users don't play a game as they optimize their own consumption schedules based on the value of the price broadcasted by the central authority. Game theoretic behavior when flexible loads are forced to bid in multi-period day ahead power markets under uniform price quantity bidding rules has been examined in [11]. Addressing wind variability, a game is formulated among various consumers to reduce supply-demand imbalance in [9].

In this paper, we model flexible loads with operational constraints and utility functions. We then formulate and solve a centralized control problem to manage the power consumption of flexible loads. Next, we formulate a decentralized problem where flexible load consumers play a game among each other. The centralized control achieves better performance than decentralized (game theoretic) control solutions in terms of the social objectives being met. The concept of "price of anarchy"

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is used to quantify the loss of efficiency in decentralized game solutions as compared to the optimal centralized control. Price of anarchy has become an interesting research topic in the communication and computer science literature [12], [13], [14], [15]. In [12], the author derived bounds on the POA for various cost sharing games. In [13], robust bounds on the POA has been derived for different smooth games. In [14], a tight bound, viz. Pigou Bound, for routing games has been computed. Johari, in [15], has shown a tight bound on the POA where a single infinitely divisible resource has been allocated among multiple competing users by proportional allocation mechanism.

In the context of smart grid, the price of anarchy has not yet been widely investigated. In [16], it has been shown that non-cooperative behavior of agents leads to suboptimal outcome with respect to co-operative behavior in dynamic oligopoly market structure. But no concrete bound on the POA has been computed. In [10], in an infinite population game, the Nash equilibrium is shown to be optimal when the charging rates of all the vehicles are equal, which may be impractical. In our previous work [17], we derived a tight lower bound on the price of anarchy for a non co-operative game problem where the objective of the control authority is to balance demand with supply in an intra-day setup.

In this paper, we prove that the non-cooperative game played among flexible load consumers for demand response is a valid monotone utility game. This is our main result. As a result we obtain a lower bound on the POA of value $1/2$. If the game does not reach Nash equilibrium (which can happen due to various reasons), we can define a weaker notion of equilibria, e.g., coarse correlated equilibria. In this case, we are able to give a bound on the efficiency of the decentralized game solution. The POA bound is robust and remains the same even for these equilibria.

II. Problem Formulation

We consider a system with consumers having flexible loads. Flexible loads are those loads whose consumption schedules can be adjusted and shifted to different time slots under some constraints. Examples are heating ventilation and air conditioning systems, pool pumps, electric vehicles, washers, dryers, refrigerators etc. Each flexible load consumer possesses a smart meter with advanced two-way communication and computational capability. The power consumption schedules of the loads will be pre-decided and controlled with the help of these meters.

A. System Model

We start with notations and assumptions:

- The time horizon is divided into T discrete slots and indexed by $t \in \mathbb{T} := (1, 2, ..., T)$.
- Flexible load consumers are denoted by $\mathbb{N} := (1, 2, ..., N)$ and indexed by i. Without loss of generality we assume that there are N flexible load consumers each having one flexible load. So the system has total N number of flexible loads.
- The power consumption of the i-th flexible load consumer at time t is denoted by $d_i(t)$.
- $d_i := (d_i(t) : t \in \mathbb{T})$ denotes the power demand vector of the i-th consumer over the whole time period T.
- $D = \{d_i : i \in \mathbb{N}\}$ denotes the set of all the consumers' power demand vectors.
- E_i denotes the feasible set for the d_i's, i.e., $d_i \in E_i$.
- Price of electricity in the system at time t is a function of total power consumption at time t and is denoted by $p(\sum_{i=1}^{N} d_i(t))$. Here we assume that the price is a convex, continuously differentiable and monotonically increasing function.
- Power transfer from load to grid is not allowed, i.e.,

$$d_i(t) \geq 0 \tag{1}$$

for all $i \in \mathbb{N}$ and $t \in \mathbb{T}$.

B. Load Model

Let $u_i(d_i)$ denote the utility function of a flexible load i in monetary units. The utility function u_i is assumed to be a positive, concave, continuously differentiable and monotonically increasing function. Depending upon the type of loads, the utility functions can be different. The utility functions for different loads have been computed in [7].

Each flexible load consumer's power consumption $d_i(t)$ satisfy the following constraints:

- The power consumption of each load is bounded above and below by d_i^{max} and d_i^{min} at each time respectively:

$$d_i^{min} \leq d_i(t) \leq d_i^{max} \tag{2}$$

where d_i^{min} is nonnegative as per (1).
- For some of the flexible loads, the total energy consumption is bounded below and above by q_i^{min} and q_i^{max} respectively:

$$q_i^{min} \leq \sum_{t=1}^{T} d_i(t) \leq q_i^{max}. \tag{3}$$

So, the set E_i can be constructed from the above constraints. As the above constraints are linear, we can write them together in the following form

$$\sum_{t=1}^{T} \gamma_i^m(t) d_i(t) \leq b^m \quad \forall m \in \mathbb{M},\ i \in \mathbb{N} \tag{4}$$

where the set $\mathbb{M} := (1, 2, ..., M)$ is the set of constraints indexed by m.

III. Centralized Control

We first consider the case where there is a central control authority that controls the overall power consumption schedule of the consumers. We assume that the authority has the price formula $p(\sum_{i=1}^{N} d_i(t))$ for that system. So it aims to maximize the total utility of the consumers while minimizing their overall cost of consumption subject to their operational constraints. This can be regarded as a theoretical ideal case against which decentralized control solutions can be compared. Thus, the control authority's objective is to

$$\underset{d_i(t)}{maximize} \quad V(D) = \sum_{i=1}^{N} u_i(d_i) - \sum_{t=1}^{T} p(\sum_{i=1}^{N} d_i(t)) \sum_{i=1}^{N} d_i(t) \tag{5}$$

subject to (4). We assume that the convex set produced by the inequality constraint functions defined by (4) is nonempty. Since this is a concave optimization problem with convex inequality constraint functions, the above assumption will ensure that global maxima exist for this problem [18]. The control authority can compute the optimal solution using the well-known KKT conditions [18]. In order to calculate these KKT conditions, let us define

$$\tilde{V} = \sum_{i=1}^{N} u_i(d_i) - \sum_{t=1}^{T} p(\sum_{i=1}^{N} d_i(t)) \sum_{i=1}^{N} d_i(t) - \sum_{i=1}^{N} \sum_{m=1}^{M} \mu_m^i (\sum_{t=1}^{T} \gamma_i^m(t) d_i(t) - b^m) \tag{6}$$

where $\lambda(t)$ and μ_m^i are the KKT multipliers. Taking partial derivatives with respect to $d_i(t)$ and $\lambda(t)$ and writing the complementary slackness condition for the inequality constraints, we obtain the following KKT conditions:

$$\frac{\partial u_i(d_i)}{\partial d_i(t)} - p(\sum_{i=1}^{N} d_i(t)) - p'(\sum_{i=1}^{N} d_i(t)) \sum_{i=1}^{N} d_i(t) - \sum_{m=1}^{M} \mu_m^i \gamma_i^m(t) = 0 \tag{7}$$

$$\mu_m^i (\sum_{t=1}^{T} \gamma_i^m(t) d_i(t) - b^m) = 0 \tag{8}$$

$$\mu_m^i \geq 0 \tag{9}$$

for all $i \in \mathbb{N}$, $t \in \mathbb{T}$, $m \in \mathbb{M}$.

If the central control authority has complete knowledge of all the flexible load consumers' utility functions and constraints, then in principle, it can compute the optimal solution using the above equations. Following points are the drawbacks of implementing centralized control approach:

1) Consumers may want to keep their utility functions and operational constraints private.
2) Consumers may like to control their loads on their own.
3) If we consider a large number of consumers, the authority may not have the computational capability of solving a high dimensional optimization problem.

So, the above centralized control is not a practical approach and consumers will control the power consumption of their loads on their own with the help of available information on their smart meters. Thus we model this scenario as a game problem in the next section.

IV. Decentralized Game Theoretic Method

There is no central authority to fully control the overall power consumption schedule. The consumers know the price function and they optimize their consumption schedules accordingly. As the price is a function of power consumption of all the consumers, we model the resulting situation as a noncooperative game. The game is defined as follows:

1) Players: Set of N consumers=$\mathbb{N}$
2) Strategy: Consumer i's strategy $d_i = (d_i(t) : t \in \mathbb{T})$
3) Payoff: For each consumer i, the payoff is to maximize

$$L_i(d_i, d_{-i}) = u_i(d_i) - \sum_{t=1}^{T} p(\sum_{i=1}^{N} d_i(t)) d_i(t) \tag{10}$$

subject to (4), where $d_{-i} = \{d_1, ..., d_{i-1}, d_{i+1}, ..., d_N\}$ is the set of power consumption vectors of all consumers other than the consumer i. We also denote the pay-off by $L_i(D)$, i.e.,

$$L_i(D) = L_i(d_i, d_{-i}). \tag{11}$$

This game is called "demand response game" and is denoted by $\mathscr{G}$. We define Nash equilibrium which is a set of all players' strategies such that no player has an incentive to deviate unilaterally. Mathematically, Nash equilibrium is the strategy d_i^* such that,

$$L_i(d_i^*, d_{-i}^*) \geq L_i(d_i, d_{-i}^*) \quad \forall i \in \mathbb{N}. \tag{12}$$

Here we assume that, each consumer will try to maximize its own payoff assuming all other consumers' strategies are fixed. This is called the "best response strategy". These strategies can be computed by solving KKT conditions for all the consumers together. In order to compute the KKT conditions, let us define

$$\tilde{L}_i = u_i(d_i) - \sum_{t=1}^{T} p(\sum_{i=1}^{N} d_i(t)) d_i(t) - \sum_{m=1}^{M} \underline{\mu}_m^i (\sum_{t=1}^{T} \gamma_i^m(t) d_i(t) - b^m) \tag{13}$$

where $\underline{\mu}_m^i$ is the KKT multiplier. Now taking partial derivatives with respect to $k_i(t)$ and writing the complementary slackness condition for the inequality constraints, we get the following KKT conditions:

$$\frac{\partial u_i(d_i)}{\partial d_i(t)} - p(\sum_{i=1}^{N} d_i(t)) - p(\sum_{i=1}^{N} d_i(t)) d_i(t) - \sum_{m=1}^{M} \underline{\mu}_m^i \gamma_i^m(t) = 0 \tag{14}$$

$$\underline{\mu}_m^i (\gamma_i^m(t) d_i(t) - b^m) = 0 \tag{15}$$

$$\underline{\mu}_m^i \geq 0 \tag{16}$$

for $t \in \mathbb{T}$, $m \in \mathbb{M}$. Now as each consumer's objective function is concave and the set produced by the inequality constraint functions is convex and compact, Nash equilibrium solution exists according to Rosen's theorem [19].

Here each consumer will calculate its own power consumption schedule solving its KKT conditions. The smart meters of all the consumers having advanced communication features will send their data to the central authority which in turn will broadcast and display the sum of the power consumption schedules to all the smart meters. Based on that data, consumers will recalculate their schedules. The process will stop until convergence or after a certain number of predefined iterations. In [8] and [10], authors studied the convergence properties of distributed algorithms. In this paper, we don't study the convergence properties of the algorithm to compute Nash equilibria, as we focus on other details to be described in the next section.

V. Demand Response Game: A Valid Monotone Utility Game

A. Definitions

Let us define a general payoff maximization game as follows.

- Set of players $\mathbb{N} = \{1,2,.....,N\}$ and is indexed by i.
- Player i's strategy vector is d_i and $d_i \in E_i$.
- $D = \{d_i : i \in \mathbb{N}\}$ denotes the set of all players' strategies.
- The payoff of a player is $L_i(D) = L_i(d_i, d_{-i})$.
- The objective function is $V(D)$ where $V : 2^D \to \mathbb{R}$ is a general function defined over all subsets of D.

A game defined in this fashion is called a valid utility game if it satisfies the following three properties [20]:

1) V is submodular , i.e., for any $A \subset A' \subset D$ and any element vector $a \in D \setminus A'$
$$V(A \cup \{a\}) - V(A) \geq V(A' \cup \{a\}) - V(A') \qquad (17)$$
2) The objective value of a player is at-least his added value for the societal objective, i.e.,
$$L_j(D) \geq V(D) - V(D - d_j) \qquad (18)$$
where d_j is the strategy vector of a player j.
3) The total value for the players is less than or equal to the total societal value, i.e.,
$$\sum_{i=1}^{N} L_i(D) \leq V(D) \qquad (19)$$

A general payoff maximization game is called a monotone game if for all $A \subseteq A' \subseteq D$, $V(A) \leq V(A')$ [20].

B. Main Result

As our main result, we will show that our demand response game is a valid monotone utility game. In section II, we have stated that utility functions are expressed in monetary units. Since the price formula is known to the consumers, we assume here the following about each consumer's utility function:

A1: The utility function of any consumer will be such that,
$$u_j(d_j) \geq \sum_{t=1}^{T} \{p(k + d_j(t))(k + d_j(t)) - p(k)k\} \qquad (20)$$

where $k = \sum_{i=1, i \neq j}^{N} d_i^{max}$. This means that the value of the utility function of a consumer will be greater than the increase in the maximum cost of power consumption in the system due to addition of that consumer for any feasible demand d_j. Now the central authority can broadcast this requirement to all the consumers as a prerequisite for participation in the demand response program. We assume that the authority have an estimate of the upper-bound of $\sum_{i=1}^{N} d_i^{max}$ which it also broadcasts to all the consumers.

Let us consider a set $A \subseteq D$. So, $A = \{d_i : i \in S\}$ where $S \subseteq \mathbb{N}$. For our problem, we already defined $V(D)$ as per (5). V is a more general function which is defined on all subsets of D. So accordingly,

$$V(A) = \sum_{i \in S} u_i(d_i) - \sum_{t=1}^{T} p(\sum_{i \in S} d_i(t))(\sum_{i \in S} d_i(t)). \qquad (21)$$

Now let us define a function

$$F(x) = p(x + a)(x + a) - p(x)x \qquad (22)$$

So,

$$F'(x) = p(x + a) + (x + a)p'(x + a) - p(x) - xp'(x) = (p(x + a) - p(x)) + (xp'(x + a) - xp'(x)) + ap'(x + a). \qquad (23)$$

Since the functions p and p' are increasing, $F'(x) \geq 0$ which means $F(x)$ is increasing for all x. So using (20), we can write

$$u_j(d_j) \geq \sum_{t=1}^{T} p(\sum_{i \in S} d_i(t) + d_j(t))(\sum_{i \in S} d_i(t) + d_j(t)) - \sum_{t=1}^{T} p(\sum_{i \in S} d_i(t))(\sum_{i \in S} d_i(t)) \qquad (24)$$

for any $d_j \in D \setminus A$. It implies that $V(A)$ is a monotonically increasing function for all $A \subseteq D$.

Next, we state the main technical result of the paper.

Theorem V.1. *Consider the demand response game defined by $\mathcal{G} = < \mathbb{N}, \{d_i\}, \{L_i(d_i, d_{-i})\} >$ with objective function $V(D)$ as defined by (5). V is a general function which is defined on all subsets of D by (21). Suppose the assumption **A1** holds true. Then this game is a valid monotone utility game.*

Proof. We show that the game $\mathcal{G}$ has the following three properties:

1. *V is submodular*

Proof: Assume two sets $A \subset A' \subset D$ and an element $d_j \in D \setminus A'$. Therefore, $A = \{d_i : i \in S\}$ and $A' = \{d_i : i \in S'\}$ where $S \subset S' \subset \mathbb{N}$.

As $d_j \notin A$

$$V(A \cup \{d_j\}) - V(A) = u_j(d_j) - \sum_{t=1}^{T} p(\sum_{i \in S} d_i(t) + d_j(t))(\sum_{i \in S} d_i(t) + d_j(t)) + \sum_{t=1}^{T} p(\sum_{i \in S} d_i(t))(\sum_{i \in S} d_i(t)) \qquad (25)$$

Similarly

$$V(A' \cup \{d_j\}) - V(A') = u_j(d_j) - \sum_{t=1}^{T} p(\sum_{i\in S'} d_i(t) + d_j(t))(\sum_{i\in S'} d_i(t) + d_j(t)) + \sum_{t=1}^{T} p(\sum_{i\in S'} d_i(t))(\sum_{i\in S'} d_i(t)) \quad (26)$$

As $p(x+a)(x+a) - p(x)x$ is a monotonically increasing function for all x, we can write,

$$p(\sum_{i\in S'} d_i(t) + d_j(t))(\sum_{i\in S'} d_i(t) + d_j(t)) + p(\sum_{i\in S'} d_i(t))(\sum_{i\in S'} d_i(t)) \geq p(\sum_{i\in S} d_i(t) + d_j(t))(\sum_{i\in S} d_i(t) + d_j(t)) + p(\sum_{i\in S} d_i(t))(\sum_{i\in S} d_i(t)) \quad (27)$$

Using (27), we compare (25) and (26) and obtain

$$V(A \cup \{d_j\}) - V(A) \geq V(A' \cup \{d_j\}) - V(A') \quad (28)$$

So, V is submodular.

2. *The objective value of a player is at-least his added value for the societal objective, i.e.,* $L_j(D) \geq V(D) - V(D - d_j)$

Proof:

$$L_j(D) = u_j(d_j) - \sum_{t=1}^{T} p(\sum_{i=1}^{N} d_i(t)) d_j(t) \quad (29)$$

and

$$V(D) - V(D - d_j) = u_j(d_j) - \sum_{t=1}^{T} p(\sum_{i=1}^{N} d_i(t))(\sum_{i=1}^{N} d_i(t)) + \sum_{t=1}^{T} p(\sum_{i=1, i\neq j}^{N} d_i(t))(\sum_{i=1, i\neq j}^{N} d_i(t)) \quad (30)$$

As $p(x)$ is an increasing function,

$$p(x) \geq p(x-a) \quad (31)$$

So,

$$p(x)x - p(x-a)(x-a) \geq p(x)a \quad (32)$$

Therefore we can write,

$$p(\sum_{i=1}^{N} d_i(t))(\sum_{i=1}^{N} d_i(t)) - p(\sum_{i=1, i\neq j}^{N} d_i(t))(\sum_{i=1, i\neq j}^{N} d_i(t)) \geq p(\sum_{i=1}^{N} d_i(t))(d_i(t)) \quad (33)$$

Thus using (33), we compare (30) and (29) and get

$$L_j(D) \geq V(D) - V(D - d_j) \quad (34)$$

3. *The total value for the players is less than or equal to the total societal value, i.e.,* $\sum_{i=1}^{N} L_i(D) \leq V(D)$

Proof: This is true by the definition of the game, i.e.,

$$\sum_{i=1}^{N} L_i(D) = \sum_{i=1}^{N} u_i(d_i) - \sum_{t=1}^{T} p(\sum_{i=1}^{N} d_i(t)) \sum_{i=1}^{N} d_i(t) = V(D) \quad (35)$$

So using assumption 1 and properties 1, 2 and 3, we conclude that the demand response game is a valid monotone utility game. □

VI. Price of Anarchy

Comparing (7)-(9) with (14)-(16), we can say that the KKT conditions of the centralized control and the demand response game are not the same. So the optimal centralized control will achieve better performance than the noncooperative game. Price of Anarchy (POA) is a concept to quantify the inefficiency of selfish behavior in a noncooperative game as compared with optimal centralized control [13]. It is defined as the worst-case ratio of the objective function value of a equilibrium solution of a game and that of a centralized optimal solution. Since in the earlier section, we have proved that our demand response game is a valid utility game, we are now in a position to obtain some interesting results regarding the lower bound of its POA.

A. Games reaching a Nash equilibrium

Let us consider a general payoff maximization game which satisfies (19) for any solution set D. This game is called a (λ, μ) smooth game if

$$\sum_{i=1}^{N} L_i(D^*) \geq \lambda V(D') - \mu V(D^*) \quad (36)$$

where D^* and D' are any two solution sets of the game. Now it is shown in [13] with the help of Theorem 3.2 of [21] that a valid monotone utility game is (1,1) smooth and the lower bound on the POA is 1/2. Since our demand response game is a valid monotone utility game, it results in the following corollary.

Corollary VI-A.1. *The demand response game* $\mathscr{G} = <\mathbb{N}, \{d_i\}, \{L_i(d_i, d_{-i})\}>$ *is a (1,1) smooth game. Moreover, the lower bound of the price of anarchy of a pure Nash equilibrium is at least 1/2.*

B. Games reaching a coarse correlated equilibrium

We have shown that Nash equilibrium for our game exists, but it is not unique. Even if the equilibria exist, there can be a number of reasons for which the players may not reach an equilibrium [13]. So, we here consider a weaker notion of equilibria i.e. coarse correlated equilibria for a game for which Nash equilibria does not exist or exist but can not be reached. Let us define a probability distribution σ_i over strategy d_i of a player for all $i \in \mathbb{N}$. So $\sigma = \Pi_{i=1}^{N} \sigma_i$ is the product probability distribution. Here a benevolent mediator is present who knows the product probability distribution. The mediator will draw an outcome D from σ and privately recommend the strategy d_i to each player. The course correlated equilibrium strategy d_i^*

of the demand response game can be defined as probability distribution over outcomes that satisfies

$$\mathbb{E}_{D\sim\sigma}(L_i(d_i^*, d_{-i}^*)) \geq \mathbb{E}_{D\sim\sigma}(L_i(d_i, d_{-i}^*)) \quad \forall i \in \mathbb{N}. \tag{37}$$

Now since the demand response game is a (1,1) smooth game, the bound derived via smoothness argument extends automatically, with no quantitative degradation to other weaker equilibria notions [13]. This is called intrinsic robustness property of the price of anarchy. So, the next corollary is as follows

Corollary VI-B.1. *Consider the demand response game* $\mathscr{G} =< \mathbb{N}, \{d_i\}, \{L_i(d_i, d_{-i})\} >$ *that reaches a course correlated equilibrium. Then*

$$\mathbb{E}_{D\sim\sigma}(V(D^*)) \geq 0.5V(D^O) \tag{38}$$

where D^* *is the course correlated equilibrium solution and* D^O *is the optimal solution of the centralized control problem.*

VII. Conclusion and Future Work

In this paper, we have investigated the flexible demand control problem in a smart grid by centralized and decentralized fashion. We show that the centralized control, though a theoretically ideal case, may be impractical. So we consider a decentralized problem where flexible load consumers play a game among each other. This solution is practical, but due to the selfish nature of consumers, the social objective will not be met optimally and there will be some efficiency loss in the system. We have proved that the demand response game considered by us is a valid monotone utility game under some assumptions. This leads to some important results regarding the lower bound of the price of anarchy. The lower bound of the POA is half for a demand response game with pure Nash equilibria. Even when the game does not reach Nash equilibria and we define a weaker notion of equilibria, i.e., coarse correlated equilibria, the lower bound on the POA remains same.

We are currently exploring several questions in this general direction:

1) How to find a tight lower bound on the price of anarchy for these types of games?
2) How to induce or incentivize co-operation among consumers in the decentralized game problem so that efficiency of the decentralized game with respect to optimal centralized problem increases?

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