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Analysis of Solar Energy Aggregation under Various Billing Mechanisms

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***Abstract*—Ongoing reductions in the cost of solar photovoltaic (PV) systems are driving their increased installations by residential households. Various incentive programs such as feed-in tariff, net metering, net purchase and sale that allow the prosumers to sell their generated electricity to the grid are also powering this trend. In this paper, we investigate sharing of PV systems among a community of households, who can also benefit further by pooling their production. Using cooperative game theory, we find conditions under which such sharing decreases their net total cost. We also develop allocation rules such that the joint net electricity consumption cost is allocated to the participants. These cost allocations are based on the cost causation principle. The allocations also satisfy the standalone cost principle and promote PV solar aggregation. We also perform a comparative analytical study on the benefit of sharing under the mechanisms favorable for sharing, namely net metering, and net purchase and sale. The results are illustrated in a case study using real consumption data from a residential community in Austin, Texas.**

***Index Terms*—Community solar, Solar PV aggregation, Net metering, Net purchase and sale, Cost allocation based on cost causation, Cooperative games.**

I. Introduction

Greater adoption of residential scale solar photovoltaic renewable electric energy is a compelling engineering and sustainability objective. Powered by (a) ongoing price reductions [1], (b) various types of subsidies [2], and (c) desire to decarbonize the energy system [3], there has been dramatic increase in rooftop solar PV installations. As residential users install rooftop solar panels and generate portions of their electricity needs, the need for fossil fuel based electric power plants decreases. The major challenge of integrating photovoltaic systems like any other renewable resources in the power system is handling the associated variability and uncertainty of generation. PV systems cannot generate electricity during the night, the generated energy varies depending on cloud shading, and this variation cannot be perfectly forecast in advance. Recently, efficient statistical approaches have been developed to quantify variability and uncertainty of PV systems [4], [5]. Implementation of demand response [6] and deployment of storage systems [7] are considered to be significant potential solutions to reduce variability of renewable integration. The total PV installations globally have reached 300 GW by 2016 [8] of which about 28% are decentralized grid connected. We are focused on developing techniques and tools that can further increase the cost-effectiveness of rooftop solar PV installations.

Three main billing programs around the world enable homeowners to sell their PV electricity to the grid: *feed-in-tariff*, *net metering*, and *net purchase and sale* [9]. Some utilities consider these programs as a threat to their business models [10]. On the other hand, socio-economic-environmental policies surrounding climate change have led various governments to encourage such programs.

In this paper, we investigate how sharing the electricity generated by rooftop PV in a cooperative manner can further facilitate their adoption by decreasing overall energy costs. We assume that the rooftop solar panels are electrically connected with each other and the necessary hardware for electricity sharing has been installed. Sharing economy has been a huge success in housing and transportation sectors in recent times [11]. It has been propelled by the desire to leverage under-utilized infrastructures in existing houses and cars. Companies like Uber, Lyft, AirBnB, VRBO made large impacts in transportation and housing sectors [12]. In the electricity sector, there is some research on modeling resource sharing. Aggregation of geographically diverse renewable energy sources have potential to reduce the variability of renewable generation [13]. Cooperation among renewable resources to develop joint *ex-ante* power contract for bidding in a two settlement market in order to reduce variability and to improve the expected profit has been developed using coalitional game theory [14]. In the same two settlement market set-up, using coalitional game theory aggregation of renewable resources to improve realized profit has been analyzed in [15]. Cooperative game theoretic analysis of multiple demand response aggregators in a virtual power plant and their cost allocation has been tackled in [16]. Sharing of storage firms under a local spot market has been analyzed using non-cooperative game theory [17].

Community solar projects, where a community scale solar energy plant is developed and shared by many households, are getting popular [18]. Various countries like Germany, Denmark, Australia, United Kingdom, United States are already investing significantly in this type of projects [19]. Benefit of shared solar PV under net metering has been studied in [20]. To the best of our knowledge, sharing of PV systems in a cooperative manner among different houses under various billing schemes has not been investigated. Can cooperation among rooftop PV systems reduce *ex-post* realized costs to

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prosumers under all billing mechanisms? If the answer is affirmative, to encourage and preserve cooperative sharing, it will be crucial to have a just and reasonable allocation of the resulting cost reduction or benefit increase to the participating individuals. These are precisely the questions we analyze in this paper.

Our results show that there is no advantage for cooperation in the case of feed-in tariff. Using cooperative game theory, we derive a necessary and sufficient condition on pricing under which cooperation is advantageous for the participating prosumers in net metering, and net purchase and sale mechanisms. We develop rules for allocating joint cost based on the cost causation principle [21]. We prove that these cost allocation rules also follow standalone cost principle – *i.e.* they are in the core of the cooperative games of net electricity consumption costs. Next, we conduct a comparative study about the benefits of cooperation under net metering, and net purchase and sale mechanisms. These results provide theoretical basis for sharing of rooftop PV generation among prosumers. We also develop a case study based on real consumption and generation data of a residential community in Austin, Texas, to illustrate our results.

The remainder of the paper is organized as follows. In Section II, we formulate the problem, introduce notation and describe the system model. In Section III, we review basic results on cooperative game theory and cost allocation based on cost causation that are used in the derivation of the results. In Section IV, we report our main results on aggregation of PV solar energy. A case study to illustrate the theoretical results is presented in Section V. Finally, we conclude our work in Section VI.

II. Problem formulation

Consider a residential community of N households with PV systems, that is represented by the set $\mathcal{N} = \{1, 2, \ldots, N\}$. The retail price of electricity consumption for the set of households is λ. They can also sell their electricity generation at price μ. Both λ and μ are decided by the utility operating under the system operators based on different factors, policies and regulations [9]. We consider three programs: feed-in tariff, net metering, and net purchase and sale that allow households to sell their generated electricity to the grid. These three mechanisms are explained in detail in [9], indicating their countries of usage and also analyzing their impact on social welfare. In the sequel, and in order to improve clarity of the presentation, we shall consider that the prices λ and μ do not change during each billing period. However, our results could be easily extended to an scenario of either time-of-use (TOU) tariffs or dynamic prices at the expense of a burdensome notation. We also assume here that the households obtain more utility than their cost for solar generation and thus rationally install solar panels.

In *feed-in tariff* program, the households can sell all of their PV generation at price μ and they must purchase all of their consumed electricity from the grid at price λ. In contrast, under the programs net metering, and net purchase and sale, electric utility of the grid purchases only the net amount of the PV generation of the households that exceeds their consumption. But the two programs compute their net electricity consumption and generation in different ways. Under *net metering* program, when the PV generation of a household exceeds its consumption, the electric meter runs backwards. At the end of a billing period, if the amount of electricity generation is more than consumption, the household is paid for the net PV generation at price μ. If the amount of electricity generation is less than consumption, the household has to pay the net amount consumed at price λ. Under *net purchase and sale* program, the generation and consumption is compared at each time and the PV generated electricity is fed into the system actually when generation exceeds consumption and purchased by the utility at price μ. Otherwise, the consumed electricity is purchased by the household at price λ. So the amount of electricity is compared moment by moment in this program instead of at the end of a billing period as in the net metering program.

Let us consider a billing period $[t_0, t_f]$ of duration $T = t_f - t_0$. For the i-th household at time $t \in [t_0, t_f]$, let the electricity consumption be $q_i(t)$, and the generation by rooftop solar panels be $g_i(t)$.

The net electricity consumption cost of the household for the entire billing period is:

$$C_i^b = \lambda Q_i^b - \mu G_i^b, \tag{1}$$

where Q_i^b and G_i^b denote the i-th household's energy consumption and generation during the billing period for the billing mechanism b, respectively. These quantities are computed in a different way depending on the billing mechanism.

For *feed-in tariff* mechanism ($b = \text{FiT}$):

$$Q_i^{\text{FiT}} = \int_{t_0}^{t_f} q_i(t)dt, \quad G_i^{\text{FiT}} = \int_{t_0}^{t_f} g_i(t)dt. \tag{2}$$

For *net metering* mechanism ($b = \text{NM}$):

$$Q_i^{\text{NM}} = \left(\int_{t_0}^{t_f} q_i(t)dt - \int_{t_0}^{t_f} g_i(t)dt \right)_+, \tag{3}$$

$$G_i^{\text{NM}} = \left(\int_{t_0}^{t_f} g_i(t)dt - \int_{t_0}^{t_f} q_i(t)dt \right)_+, \tag{4}$$

where $(\cdot)_+$ denotes the positive part function – *i.e.* $(x)_+ = \max\{x, 0\}$ for any real number x.

For *net purchase and sale* mechanism ($b = \text{NPS}$):

$$Q_i^{\text{NPS}} = \int_{t_0}^{t_f} (q_i(t) - g_i(t))_+ dt, \tag{5}$$

$$G_i^{\text{NPS}} = \int_{t_0}^{t_f} (g_i(t) - q_i(t))_+ dt. \tag{6}$$

Note that the net consumption for each household $i \in \mathcal{N}$ is obtained as $D_i = Q_i^b - G_i^b$ and its value is independent of the billing mechanism, – *i.e.* it has the same value for the three billing mechanisms. This follows by the properties of the positive part function. Note that for any pair x, y of real numbers $x - y = (x - y)_+ - (y - x)_+$. Then, on one hand

$$\begin{aligned} Q_i^{\text{FiT}} - G_i^{\text{FiT}} &= (Q_i^{\text{FiT}} - G_i^{\text{FiT}})_+ - (G_i^{\text{FiT}} - Q_i^{\text{FiT}})_+ \\ &= Q_i^{\text{NM}} - G_i^{\text{NM}}. \end{aligned}$$

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On the other hand:

$$q_i(t) - g_i(t) = (q_i(t) - g_i(t))_+ - (g_i(t) - q_i(t))_+,$$

and integrating the last expression in the billing interval $[t_0, t_f]$, we obtain $Q_i^{\text{FiT}} - G_i^{\text{FiT}} = Q_i^{\text{NPS}} - G_i^{\text{NPS}}$. Thus, $D_i = Q_i^{\text{FiT}} - G_i^{\text{FiT}} = Q_i^{\text{NM}} - G_i^{\text{NM}} = Q_i^{\text{NPS}} - G_i^{\text{NPS}}$. However, the cost C_i^b is different for each billing mechanism $b \in \{\text{FiT}, \text{NM}, \text{NPS}\}$.

Let $\mathcal{S} \subseteq \mathcal{N}$ denote a coalition of households that decide to cooperate to share their electricity generation by rooftop solar panels and save electricity costs. We assume that the rooftop solar panels are electrically connected with each other and the houses also have necessary hardwares for electricity sharing. The cost of the necessary electrical connection is a fixed cost, similar to acquiring the PV systems. Since this is not an operational cost, but an investment cost, we have not included it in our operational model.

The total energy consumption and generation of the coalition for the interval $[t_0, t_f]$ and the billing mechanism b are denoted as $Q_{\mathcal{S}}^b$ and $G_{\mathcal{S}}^b$, respectively. The consumption is charged at price λ and the generation is paid at price μ, which are assumed to be constant during the billing period. Consequently, the cost of the coalition for the time interval $[t_0, t_f]$ is

$$C_{\mathcal{S}}^b = \lambda Q_{\mathcal{S}}^b - \mu G_{\mathcal{S}}^b, \tag{7}$$

where the expressions of $Q_{\mathcal{S}}^b$ and $G_{\mathcal{S}}^b$ depend on the billing mechanism $b \in \{\text{FiT}, \text{NM}, \text{NPS}\}$.

Cooperation is advantageous for the set $\mathcal{N}$ of households under the billing mechanism b if the joint electricity cost $C_{\mathcal{S}}^b$ is a subadditive function, – *i.e.* if for any any pair of disjoint coalitions $\{(\mathcal{S}, \mathcal{T}) : \mathcal{S}, \mathcal{T} \subseteq \mathcal{N}\}$, $C_{\mathcal{S}\cup\mathcal{T}}^b \leq C_{\mathcal{S}}^b + C_{\mathcal{T}}^b$. We are interested in studying under which billing mechanisms and what conditions, cooperation is advantageous. Moreover, if cooperation produce some benefit, we want to develop mechanisms to allocate it among the households in a way that is satisfactory for all of them. These questions can be formally answered using cooperative game theory.

III. Background on Cooperative Game Theory for Cost Sharing and Allocation

Game theory deals with rational behavior of economic agents in a mutually interactive setting [22]. Cooperative games (or coalitional games) have been used extensively in diverse disciplines such as social science, economics, philosophy, psychology and communication networks [23], [24]. Here, we focus on cooperative games for cost sharing [25]. We explain the theory with the setup and notation of our solar PV aggregation problem.

Let $\mathcal{N} := \{1, 2, \dots, N\}$ denote a finite collection of players. In a cooperative game for cost sharing, the players want to minimize their joint cost and share the resulting cost cooperatively.

Definition 1 (Coalition): A *coalition* is any subset $\mathcal{S} \subseteq \mathcal{N}$. The number of players in a coalition $\mathcal{S}$ is denoted by its cardinality, $|\mathcal{S}|$. The *set of all possible coalitions* is defined as the power set of $\mathcal{N}$, $2^{\mathcal{N}} = \{\mathcal{S} : \mathcal{S} \subseteq \mathcal{N}\}$. The *grand coalition* is the set of all players $\mathcal{N}$, and the *empty coalition* is the empty set $\varnothing$.

Definition 2 (Game and Cost): A cooperative game is defined by a pair $(\mathcal{N}, C)$ where $C : 2^{\mathcal{N}} \to \mathbb{R}$ is the *cost function* that assigns a real value to each coalition $\mathcal{S} \subseteq \mathcal{N}$ and $C(\mathcal{S}) = 0$ if $\mathcal{S} = \varnothing$.
Here, the *cost of the coalition* $\mathcal{S}$ is denoted by $C_{\mathcal{S}} = C(\mathcal{S})$. We also denote by $C_i = C(\{i\})$ the cost of the agent i – *i.e.* a singleton coalition.

Definition 3 (Subadditive Game): A cooperative game $(\mathcal{N}, C)$ is *subadditive* if, for any pair of disjoint coalitions $\mathcal{S}, \mathcal{T} \subseteq \mathcal{N}$, we have $C_{\mathcal{S}} + C_{\mathcal{T}} \geq C_{\mathcal{S}\cup\mathcal{T}}$.

Here we consider the value of the coalition $C_{\mathcal{S}}$ is *transferable* among players. The central question for a subadditive cost sharing game with transferrable value is how to *fairly* distribute the coalition value among the coalition members.

Definition 4 (Cost Allocation): A *cost allocation* for the coalition $\mathcal{S} \subseteq \mathcal{N}$ is a vector $x \in \mathbb{R}^N$ whose entry x_i represents the allocation to member $i \in \mathcal{S}$ ($x_i = 0, \quad i \notin \mathcal{S}$).

Definition 5 (Imputation): A cost allocation x for the grand coalition $\mathcal{N}$ is said to be an *imputation* if it is simultaneously budget balanced – *i.e.* $C_{\mathcal{N}} = \sum_{i=1}^{N} x_i$, and individually rational – *i.e.* $C_i \geq x_i, \forall i \in \mathcal{N}$. Let $\mathcal{I}$ denote the set of all imputations.

The fundamental solution concept for cooperative games is the *core* [22].

Definition 6 (The Core): The *core* $\mathcal{C}$ for the cooperative game $(\mathcal{N}, C)$ with *transferable cost* is defined as the set of cost allocations such that no coalition can have cost which is lower than the sum of the members current costs under the given allocation.

$$\mathcal{C} := \left\{ x \in \mathcal{I} : C_{\mathcal{S}} \geq \sum_{i\in\mathcal{S}} x_i, \ \forall \mathcal{S} \in 2^{\mathcal{N}} \right\}. \tag{8}$$

A classical result in cooperative game theory, known as Bondareva-Shapley theorem, gives a necessary and sufficient condition for a game to have nonempty core. To state this theorem, we need the following definition.

Definition 7 (Balanced Game and Balanced Map): A cooperative game $(\mathcal{N}, C)$ for cost sharing is *balanced* if for any balanced map α, $\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S}) C_{\mathcal{S}} \geq C_{\mathcal{N}}$ where the map $\alpha : 2^{\mathcal{N}} \to [0, 1]$ is said to be *balanced* if for all $i \in \mathcal{N}$, we have $\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S}) \mathbf{1}_{\mathcal{S}}(i) = 1$, where $\mathbf{1}_{\mathcal{S}}$ is the indicator function of the set $\mathcal{S}$, – *i.e.* $\mathbf{1}_{\mathcal{S}}(i) = 1$ if $i \in \mathcal{S}$ and $\mathbf{1}_{\mathcal{S}}(i) = 0$ if $i \notin \mathcal{S}$.
Next we state the Bondareva-Shapley theorem.

Theorem 1 (Bondareva-Shapley Theorem [24]): A coalitional game has a nonempty core if and only if it is balanced.

There are a number of solutions that exist for a cooperative game. The most prominent are the *Shapley value*, the *nucleolus* [23] and the minimum worst-case excess allocation [14]. If a game is balanced, the nucleolus and the minimum worst-case excess are always in the core. For a concave cost sharing game, the Shapley value is also in the core. These allocations are computationally costly for large number of players. In order to circumvent this problem and also to analyse properties of an allocation, we proposed in [21] an axiomatic framework to

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characterize *just and reasonable* cost allocation rules. These axioms are: *equity*, *monotonicity*, *individual rationality*, *budget balance*, and *standalone cost principle*. These axioms are established using a variable that characterizes the cause of cost of an agent. For the problem of allocation of the aggregated cost of a group of households, the net individual consumption $D_i = Q_i^b - G_i^b$ for $i \in \mathcal{N}$ is a natural choice for the variable that characterizes the cause of cost. Then, we establish the following axioms.

Axiom 1 (Equity): If two agents i and j have same net consumptions, the allocated costs must be the same – *i.e.* if $D_i = D_j$ then $x_i = x_j$.

Axiom 2 (Monotonicity): If two agents i and j have net consumptions of the same sign, and agent i has a higher net consumption than agent j, then the absolute value of the allocated cost to i must be higher than the absolute value of the allocated cost to j – *i.e.* if $D_iD_j \geq 0$ and $|D_i| \geq |D_j|$ then $|x_i| \geq |x_j|$.

Axiom 3 (Individual Rationality): The allocated cost must be less than the cost if the agent would not have joined the aggregation – *i.e.* $x_i \leq C_i$.

Axiom 4 (Budget Balance): The cost allocation rule would be such that the sum of allocated costs must be equal to the net electricity consumption cost – *i.e.* $\sum_{i\in\mathcal{N}} x_i = C_{\mathcal{N}}$.

Axiom 5 (Standalone Cost Principle): For every coalition $\mathcal{S} \subseteq \mathcal{N}$, we have $\sum_{i\in\mathcal{S}} x_i \leq C_{\mathcal{S}}$.

From Axiom 5 and Definition 6, it is easy to see that an allocation that satisfies the standalone cost principle belongs to the core of the cooperative game of cost sharing.

A cost causation based allocation was inspired by the tariffs proposed by Kirby in [26] and should follow the axioms of equity, monotonicity, individual rationality and budget balance, but not necessarily the standalone cost principle. However, not every allocation rule satisfying these four axioms follows the cost causation principle, because they do not explicitly take into account whether agents are causing or mitigating costs.

Using again the net individual consumption D_i for $i \in \mathcal{N}$ as the variable signaling cause or mitigation of cost, we state the following cost causation axioms.

Definition 8 (Cost Causation and Mitigation): Let $D_{\mathcal{N}}$ be the net consumption of a group $\mathcal{N}$ of agents. It is said that agent i is causing cost if $D_i > 0$, and is mitigating cost if $D_i < 0$.

Based on this definition, we introduce two new cost causation based axioms: *penalty for cost causation* and *reward for cost mitigation*.

Axiom 6 (Penalty for Causing Cost): Those individuals causing cost should pay for it, – *i.e.* $x_i > 0$ for any $i \in \mathcal{N}$ such that $D_i > 0$.

Axiom 7 (Reward for Mitigating Cost): Those individuals mitigating cost should be rewarded, – *i.e.* $x_i < 0$ for any $i \in \mathcal{N}$ such that $D_i < 0$.

Using the previously introduced axioms, a *cost causation based* allocation rule is formally defined as follows.

Definition 9 (Cost Causation based Allocation Rule): A cost allocation rule is said to be a *cost causation based allocation* rule if it satisfies Axioms 1–4 and 6–7.

It is interesting to remark that some well-known allocation rules such as the *proportional rule*, or the *Shapley value*, do not satisfy the cost causation axioms [21].

IV. Main results on solar energy aggregation and cost allocation under different billing mechanisms

In this section, we present the main results about cooperation of households for different billing programs.

A. Feed-in Tariff

For the coalition $\mathcal{S} \subseteq \mathcal{N}$, under the feed-in tariff program, the net consumption during the billing period $[t_0, t_f]$ is $D_{\mathcal{S}} = Q_{\mathcal{S}}^{\text{FiT}} - G_{\mathcal{S}}^{\text{FiT}}$, where $Q_{\mathcal{S}}^{\text{FiT}} = \sum_{i\in\mathcal{S}} Q_i^{\text{FiT}}$, $G_{\mathcal{S}}^{\text{FiT}} = \sum_{i\in\mathcal{S}} G_i^{\text{FiT}}$, and Q_i^{FiT}, G_i^{FiT} are given by equations (2).

Let us consider two disjoint coalitions $\mathcal{S}$ and $\mathcal{T}$. For this billing program, the cost of the coalition $\mathcal{S}$ is

$$C_{\mathcal{S}}^{\text{FiT}} = \lambda Q_{\mathcal{S}}^{\text{FiT}} - \mu G_{\mathcal{S}}^{\text{FiT}}. \tag{9}$$

It is easy to see that $C_{\mathcal{S}}^{\text{FiT}} + C_{\mathcal{T}}^{\text{FiT}} = C_{\mathcal{S}\cup\mathcal{T}}^{\text{FiT}}$. So cooperation is neutral and there is no advantage or harm in cooperating and sharing the PV generation. This result suggests that FiT will not encourage cooperation.

B. Net Metering

The aggregated consumption and generation for any coalition $\mathcal{S} \subseteq \mathcal{N}$ during the billing period $[t_0, t_f]$ under the net metering program are given by

$$Q_{\mathcal{S}}^{\text{NM}} = \left(\sum_{i\in\mathcal{S}} \int_{t_0}^{t_f} q_i(t)dt - \sum_{i\in\mathcal{S}} \int_{t_0}^{t_f} g_i(t)dt\right)_+, \tag{10}$$

$$G_{\mathcal{S}}^{\text{NM}} = \left(\sum_{i\in\mathcal{S}} \int_{t_0}^{t_f} g_i(t)dt - \sum_{i\in\mathcal{S}} \int_{t_0}^{t_f} q_i(t)dt\right)_+. \tag{11}$$

The net consumption of the coalition is $D_{\mathcal{S}} = Q_{\mathcal{S}}^{\text{NM}} - G_{\mathcal{S}}^{\text{NM}}$, and the joint cost of the net consumption for the coalition is

$$C_{\mathcal{S}}^{\text{NM}} = \lambda Q_{\mathcal{S}}^{\text{NM}} - \mu G_{\mathcal{S}}^{\text{NM}}. \tag{12}$$

Let $(\mathcal{N}, C^{\text{NM}})$ denote the cooperative game for the set of of households under the net metering billing program. In the following theorem, we obtain a necessary and sufficient condition for the game $(\mathcal{N}, C^{\text{NM}})$ to be subadditive and balanced.

Theorem 2: The cooperative game $(\mathcal{N}, C^{\text{NM}})$ is subadditive and balanced if and only if $\lambda \geq \mu$.

Proof: See Appendix. □

Remark 1: The condition $\lambda \geq \mu$ developed in Theorem 2 requires that the selling price of the generated electricity is always less that the retail price of the electricity. This price condition is indeed realistic because the price of consuming electricity should include generation, transmission and distribution costs. But transmission and distribution costs should not be included in the price that the utility pays for the energy generated by a prosumer having solar PV systems [27].

Remark 2: As the cooperative game is subadditive and balanced, there exists a cost allocation of the aggregated cost that is satisfactory for every member of the grand coalition. However, the existence of such an allocation does not imply that it can be easily computed.

In our case, it is possible to develop an analytical formula for a cost allocation. The aggregated cost of the grand coalition is given by

$$C_{\mathcal{N}}^{\text{NM}} = \begin{cases} \lambda Q_{\mathcal{N}}^{\text{NM}}, & \text{if } D_{\mathcal{N}} \geq 0, \\ -\mu G_{\mathcal{N}}^{\text{NM}}, & \text{if } D_{\mathcal{N}} < 0. \end{cases}$$

Note that the aggregated net consumption satisfies

$$D_{\mathcal{N}} = \sum_{i \in \mathcal{N}} D_i = \begin{cases} Q_{\mathcal{N}}^{\text{NM}}, & \text{if } D_{\mathcal{N}} \geq 0, \\ -G_{\mathcal{N}}^{\text{NM}}, & \text{if } D_{\mathcal{N}} < 0. \end{cases}$$

Thus, the cost of the grand coalition can be split up among the individual agents and, as a result, we get the following allocation.

Allocation 1 (Net Metering): The allocated cost during the billing interval $[t_0, t_f]$ for the i-th household of a residential community $\mathcal{N}$ that share energy under the net metering program is given by

$$x_i^{\text{NM}} = \begin{cases} \lambda D_i, & \text{if } D_{\mathcal{N}} \geq 0, \\ \mu D_i, & \text{if } D_{\mathcal{N}} < 0, \end{cases} \tag{13}$$

for every $i \in \mathcal{N}$.

Next, we show that Allocation 1 has some nice properties,

Theorem 3: The allocation defined by (13) is a cost causation based allocation.

Proof: See Appendix. □

In addition, Allocation 1 is a cost allocation in the core of the cooperative game. The result is stated as the following theorem.

Theorem 4: The allocation defined by (13) satisfies standalone cost principle.

Proof: See Appendix. □

C. Net Purchase and Sale

The cost of the aggregated net consumption of a coalition $\mathcal{S} \subseteq \mathcal{N}$ during the billing period $[t_0, t_f]$ under the net purchase and sale program is

$$C_{\mathcal{S}}^{\text{NPS}} = \int_{t_0}^{t_f} C_{\mathcal{S}}^{\text{NPS}}(t) dt. \tag{14}$$

The instantaneous cost function $C_{\mathcal{S}}^{\text{NPS}}(t)$ is defined as follows:

$$C_{\mathcal{S}}^{\text{NPS}}(t) = \lambda Q_{\mathcal{S}}^{\text{NPS}}(t) - \mu G_{\mathcal{S}}^{\text{NPS}}(t), \tag{15}$$

where

$$Q_{\mathcal{S}}^{\text{NPS}}(t) = (\sum_{i \in \mathcal{S}} q_i(t) - \sum_{i \in \mathcal{S}} g_i(t))_+, \tag{16}$$

$$G_{\mathcal{S}}^{\text{NPS}}(t) = (\sum_{i \in \mathcal{S}} g_i(t) - \sum_{i \in \mathcal{S}} q_i(t))_+, \tag{17}$$

and the instantaneous net consumption at time t is $D_{\mathcal{S}}(t) = Q_{\mathcal{S}}^{\text{NPS}}(t) - G_{\mathcal{S}}^{\text{NPS}}(t)$.

The net purchase and sale is equivalent to the net metering where the billing period reduces to the time instant $t \in [t_0, t_f]$. Thus, for this billing mechanism, the study has to be accomplished instantaneously, instead of after completing the billing period $[t_0, t_f]$. Consequently, the definitions and axioms of Section III are also valid, but they should be considered for each time instant $t \in [t_0, t_f]$.

Let $(\mathcal{N}, C^{\text{NPS}})$ denote the cooperative game for the set of households and the billing period $[t_0, t_f]$ under the net purchase and sale program. A necessary and sufficient condition for the successful cooperation of the group of households is given in the following theorem.

Theorem 5: The cooperative game $(\mathcal{N}, C^{\text{NPS}})$ is subadditive and balanced if and only if $\lambda \geq \mu$.

Proof: See Appendix.

Motivated by the properties of Allocation 1 for the net metering case, we propose the following cost allocation for the net purchase and sale program.

Allocation 2 (Net Purchase and Sale): The allocated cost during the the billing time interval $[t_0, t_f]$ for the i-th household of a residential community $\mathcal{N}$ that share energy under the net purchase and sale program in given by

$$x_i^{\text{NPS}} = \int_{t_0}^{t_f} x_i^{\text{NPS}}(t) dt, \tag{18}$$

where

$$x_i^{\text{NPS}}(t) = \begin{cases} \lambda D_i(t), & \text{if } D_{\mathcal{N}}(t) \geq 0, \\ \mu D_i(t), & \text{if } D_{\mathcal{N}}(t) \leq 0. \end{cases} \tag{19}$$

The properties of Allocation 2 are established in the following theorem.

Theorem 6: The allocation defined by equations (18)–(19) is a cost causation based allocation that belongs to the core of the cooperative game $(\mathcal{N}, C^{\text{NPS}})$.

Proof: See Appendix.

D. Comparative Study of Billing Mechanisms

We proved that net metering, and net purchase and sale mechanisms promote solar PV aggregation. Next, we are interested in quantifying which of these mechanisms is better from the point of view of the total cost that the prosumers pay and save as a result of sharing.

We show that the prosumers pay less under a net metering program than under a net purchase and sale program with the same prices λ and μ.

Theorem 7: Let $\mathcal{S} \subseteq \mathcal{N}$ be any coalition of prosumers. Assume that λ and μ with $\lambda \geq \mu$ are the price of purchasing and selling electricity. The cost of the electricity consumption of the coalition is less under a net metering program than under a net purchase and sale program, and the difference is given by

$$C_{\mathcal{S}}^{\text{NPS}} - C_{\mathcal{S}}^{\text{NM}} = \begin{cases} (\lambda - \mu) G_{\mathcal{S}}^{\text{NPS}}, & \text{if } D_{\mathcal{S}} \geq 0, \\ (\lambda - \mu) Q_{\mathcal{S}}^{\text{NPS}}, & \text{if } D_{\mathcal{S}} < 0. \end{cases} \tag{20}$$

Proof: See Appendix. □

The result is valid for any coalition and therefore it applies also to individual prosumers. Next, we are interested in studying if one of these programs is better in savings than the other under aggregation. We show that, under certain conditions, the net purchase and sale program produces a larger cost savings than the net metering program.

The cost saving that household $i \in \mathcal{N}$ obtains by aggregating with the other households in the community $\mathcal{N}$ for the billing period $[t_0, t_f]$ is

$$S_i^{\mathrm{NM}} = C_i^{\mathrm{NM}} - x_i^{\mathrm{NM}} = \begin{cases} 0, & \text{if } D_i D_{\mathcal{N}} \geq 0, \\ (\lambda - \mu)|D_i|, & \text{if } D_i D_{\mathcal{N}} < 0. \end{cases} \quad (21)$$

An analogous computation for the net purchase and sale program provides the instantaneous cost saving that the household $i \in \mathcal{N}$ obtains at time instant $t \in [t_0, t_f]$ by aggregation:

$$S_i^{\mathrm{NPS}}(t) = C_i^{\mathrm{NPS}}(t) - x_i^{\mathrm{NPS}}(t) = \begin{cases} 0, & \text{if } D_i(t) D_{\mathcal{N}}(t) \geq 0, \\ (\lambda - \mu)|D_i(t)|, & \text{if } D_i(t) D_{\mathcal{N}}(t) < 0. \end{cases} \quad (22)$$

For each household $i \in \mathcal{N}$, we define the sets $\Omega_i^+ = \{t \in [t_0, t_f] : D_i(t) D_{\mathcal{N}}(t) \geq 0\}$, $\Omega_i^- = \{t \in [t_0, t_f] : D_i(t) D_{\mathcal{N}}(t) < 0\}$

Then the cost saving that the household $i \in \mathcal{N}$ obtains by aggregation for the billing period $[t_0, t_f]$ is

$$S_i^{\mathrm{NPS}} = \int_{\Omega_i^+} S_i^{\mathrm{NPS}}(t) dt + \int_{\Omega_i^-} S_i^{\mathrm{NPS}}(t) dt = (\lambda - \mu) \int_{\Omega_i^-} |D(t)| dt. \quad (23)$$

The difference in the cost savings for the household $i \in \mathcal{N}$ under the two billing mechanisms is given by

$$S_i^{\mathrm{NPS}} - S_i^{\mathrm{NM}} = \begin{cases} (\lambda - \mu) \int_{\Omega_i^-} |D(t)| dt, & \text{if } D_i D_{\mathcal{N}} \geq 0, \\ (\lambda - \mu) \left(\int_{\Omega_i^-} |D(t)| dt - |D_i| \right), & \text{if } D_i D_{\mathcal{N}} < 0. \end{cases} \quad (24)$$

Using the equations (21) and (24), we obtain that $S_i^{\mathrm{NPS}} \geq S_i^{\mathrm{NM}} = 0$ for each billing period where $D_i D_{\mathcal{N}} \geq 0$. This means that, in this case, cooperation is only advantageous under the net purchase and sale program. However, for a billing period where the individual net consumption and the net consumption of the grand coalition have opposite sign – *i.e.* $D_i D_{\mathcal{N}} \leq 0$, we cannot say anything about which program produces a greater cost saving for a household, and it depends on the particular consumption patterns. In a practical scenario considering household's electricity requirements, rooftop solar capacity, solar insolation, etc., for most of the billing periods, the households have positive individual net consumptions – *i.e.* they consume more electricity than they generate. As a result, the grand coalition also has a positive net consumption. Thus, for most of the billing periods and most of the households, the net purchase and sale program produce greater cost savings than the net metering program.

E. Billing Mechanisms Around The World

Different countries use different billing mechanisms to promote solar PV adoption [9]. Germany adopted feed-in-tariff, which is not advantageous for prosumers with sharing PV systems. Most states in the USA use net metering with $\lambda = \mu$. Japan uses net purchase and sale but in their case $\mu \geq \lambda$. None of these billing mechanisms are advantageous for sharing. A high selling price $\mu \geq \lambda$ makes solar PV very attractive for prosumers because it produces large reduction in electricity bills. However, this policy is harmful for utilities that have to buy the generated energy by the households at a higher price than the market price as it includes transmission and distribution costs [27]. It is expected that as the number of households with rooftop solar PV increases, the selling price of the PV generated electricity approaches the price that the utility pays for buying electricity in the market minus the transmission and distribution costs. Thus, in future, μ will be less than the retail price of electricity λ, and this will promote aggregation as a means to reduce consumption cost and integrate more renewable energy by taking advantage of the solar PV excess.

Recently, legislators of Nevada introduced a new compensation [28] for excess solar energy exported to the grid at 95 percent of the retail electricity rate. For every 80 megawatts of solar deployed, the export credit is set to decline by 7 percent, to a floor of 75 percent of the retail rate. NV Energy also sought to have the 95 percent calculation apply to every kilowatt-hour of energy a prosumer exports to the grid, rather than the monthly netting amount, a change that decreases the value prosumers get for the energy they export. It is favorable for cooperation as λ is greater than μ. Also the scheme reduces the value for the prosumer as calculations does not wait for monthly netting amount, rather depends upon some energy consumption. The mechanism is closer to net purchase and sale and as per our analysis in this section, the sharing PV systems in this mechanism might probably give more overall advantage to prosumers compared to net metering, though the real advantage will depend on net consumption of a particular prosumer, as well as that of the whole cooperative group.

V. A CASE STUDY

We consider a community of 80 households, located in a residential area of Austin, TX, that have PV rooftop panels and decide to share their generation. The consumption and generation data have been obtained from the Pecan Street project [29]. The codes of the 80 prosumers selected for this study are given in Table I. For each prosumer we have retrieved real data of power consumption and solar power generation for every 15 minutes. The period under study is the complete year of 2016, and the billing period is one month. As we have analytically proved that there is no advantage of sharing under feed-in tariff, in this study, we only analyze the impact on cost and savings in sharing PV systems under net metering, and net purchase and sale.

A. Data Analysis

We have considered $\lambda = 11.02$¢/kWh and $\mu = 0.57\lambda$. The value of λ is the average retail price of electricity in Texas during 2016, obtained from the Energy Information Administration (EIA) Data Browser [30] while the value of μ corresponds exclusively to the generation part of the retail price of electricity in the US – *i.e.* after discounting the transmission and distribution costs. This figure has been

TABLE I
CODES OF THE 80 PROSUMERS USED IN THE STUDY

Prosumers' Codes									
26	77	93	171	370	379	545	585	624	744
781	890	1283	1415	1697	1792	1800	2072	2094	2129
2199	2233	2557	2818	2925	2945	2980	3044	3310	3367
3456	3482	3538	3649	4154	4352	4373	4447	4767	4874
5035	5129	5218	5357	5403	5658	5738	5785	5874	5892
6061	6063	6578	7024	7030	7429	7627	7719	7793	7940
7965	7989	8046	8059	8086	8156	8243	8419	8645	8829
8995	9001	9134	9235	9248	9647	9729	9937	9971	9982

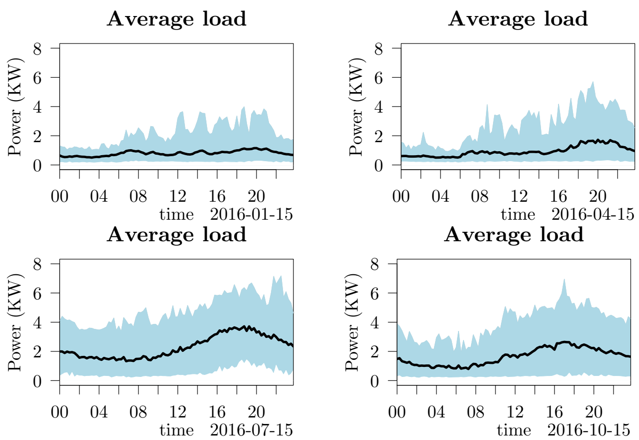

Fig. 1. Daily average load

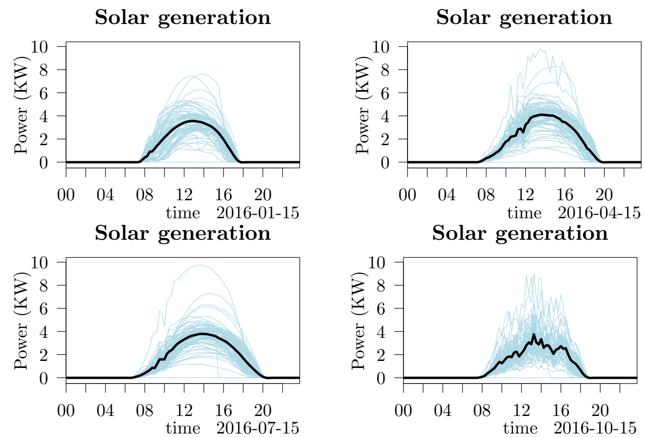

Fig. 2. Daily Solar PV generation

TABLE II
COMMUNITY MONTHLY TOTAL CONSUMPTION AND GENERATION

Month	**(a)**	**(b)**	**(a)–(b)**
1	56 807.87	44 503.73	12 304.14
2	48 200.62	52 105.83	−3905.21
3	52 714.26	52 944.47	−230.21
4	60 270.83	51 398.36	8872.47
5	77 184.61	48 118.61	29 066.00
6	113 583.74	61 418.20	52 165.54
7	134 202.32	66 716.79	67 485.52
8	119 990.42	54 610.72	65 379.69
9	109 313.42	54 128.38	55 185.04
10	83 020.00	53 773.55	29 246.45
11	55 200.10	33 601.74	21 598.36
12	61 193.50	25 028.61	36 164.89
Total	971 681.68	598 348.98	373 332.69

(a) Energy Consumption kW h
(b) Energy Generation kW h

obtained using the average costs of electricity transmission and distribution in the US in 2016 according to the *Annual Energy Outlook 2017* [31]. These prices corresponds to the implicit assumption that the prosumers are in a competitive environment where the utility distribution companies buy electricity at the market price of generation.

In Figure 1, we show the daily average consumption per household in black solid line. The shaded light blue area represents the interval between the 5% and 95% quantiles of the consumption distribution of the community prosumers. For this analysis of daily generation and consumption, since it is not possible to show here every day of the year, we have chosen four specific dates, each one in a different season. These dates are 2016-01-01, 2016-04-01, 2016-07-01 and 2016-10-01. Since each prosumer has PV rooftop panel, the daily average solar power generation per prosumer is depicted in Figure 2 in black solid line for the same specific dates. In addition, the light blue solid lines are the power generation curves for every prosumer in the community.

The total electricity consumption of the community during 2016 is 971 681 MW h and the total generation is 598 349 MW h. The monthly total consumption and generation for the 80 prosumers is given in Table II. Notice that consumption is larger in summer and fall months because of the use of air conditioners. In these months the solar generation also increases but not at the same rate as that of the consumptions. The consumption increase from January to July is 236% while the generation increase is only 150%. The net consumption of the community in February and March is negative. The average monthly consumption and generation per prosumer is 1012.17 kW h and 623.28 kW h, respectively.

The distribution of the monthly community consumption and generation are depicted in Figures 3 and 4, respectively. We show two plots for each figure. The first plot represents the distribution of consumption and generation per prosumer, while the second one is the distribution of consumption and generation per month. The average value is represented by a solid black line with round marks. The interquantile interval between 5% and 95% is shown as a light blue bar. The remaining 10% cases are shown as dark blue bars. We can see that most residents have monthly energy consumptions near the average value with some seasonal variation. The consumption is higher during summer and lower during spring. There are two residents (codes 5357 and 9647 corresponding to positions 44 and 76 in the horizontal axis) with average monthly consumptions near to 3000 kW h, much higher than the rest of residents. The plots of the rooftop PV solar generation distribution show that the monthly generation is near to the average value of 623.28 kW h with some seasonal variation. As it is expected, the generation is higher during June and July because there are more insolation hours and lower in November and December.

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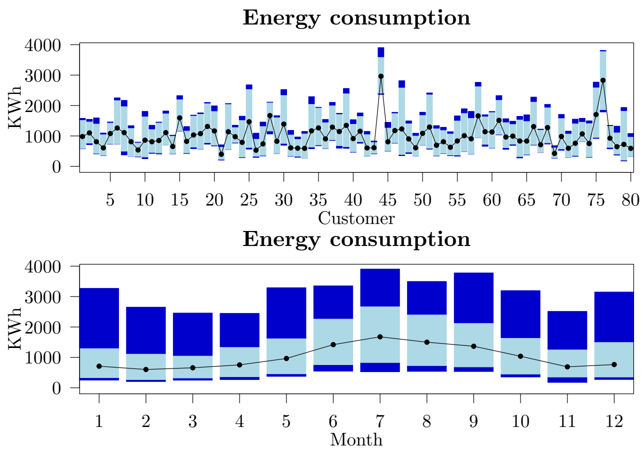

Fig. 3. Community load

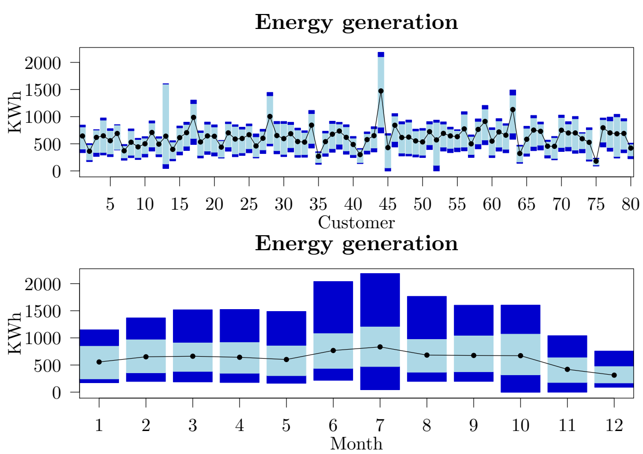

Fig. 4. Community PV solar generation

TABLE III
SUMMARY OF COST SAVINGS FOR NET METERING (I)

Month	(a)	(b)	(c)	(d)
1	1570.02	1355.92	214.11	13.64
2	136.45	−245.30	381.75	279.78
3	404.29	−14.46	418.75	103.58
4	1253.37	977.75	275.62	21.99
5	3289.97	3203.07	86.90	2.64
6	5765.62	5748.64	16.97	0.29
7	7444.56	7436.90	7.66	0.10
8	7209.74	7204.84	4.89	0.07
9	6106.10	6081.39	24.71	0.40
10	3358.26	3222.96	135.30	4.03
11	2445.12	2380.14	64.98	2.66
12	3990.26	3985.37	4.89	0.12
Total	42 973.74	41 337.22	1636.52	3.96

(a) Cost without sharing ($) (b) Cost with sharing ($)
(c) Cost savings (a)–(b) ($) (d) Cost savings (c)/|(a)| (%)

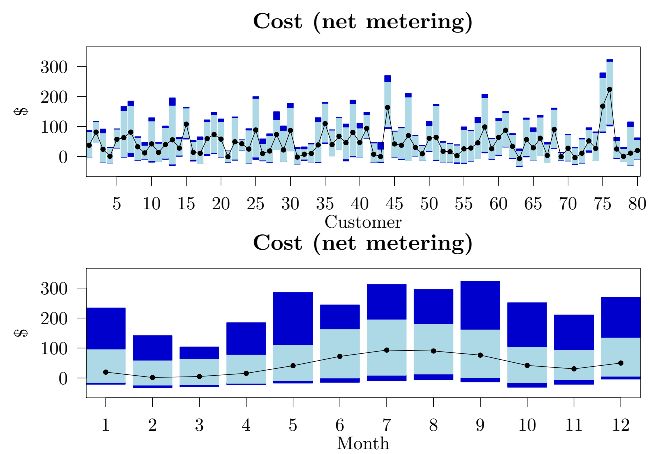

Fig. 5. Community cost for net metering billing mechanism

B. Results

We analyze now the cost of energy consumption for each resident depending on the billing mechanism.

In both the cases, we show a table with the sum of the costs for the 80 households for each month and a figure with the distribution of the monthly cost of electricity. We include two plots in this figure. The first plot represents the monthly cost distribution per prosumer, while the second one is the prosumer cost per month. The average value is represented by a solid black line with round marks. The interquantile interval between 5% and 95% is shown as a light blue bar. The remaining 10% cases are shown as dark blue bars.

In Table III we show the costs for the net metering billing mechanism. In this case, the total annual cost for the community is $42 973.74, corresponding to a monthly average per household of $44.76. If the residents share their solar rooftop generation and allocate the costs according to Allocation 1 (13), the total annual cost is $41 337.22 corresponding to a monthly average cost per household of $43.10 and a cost reduction of 3.96%.

The distribution of the monthly costs and savings are depicted in Figures 5 and 6. In Figure 5 we show the monthly cost distribution, while in Figure 5 we show the monthly cost savings distribution. Note that the higher cost savings are obtained in February and March. The reason is that the net consumption in these months is negative, see Table II.

In Table IV, we show the savings for the households. Due to space limitation we only show the 20 prosumers that obtain higher annual savings. They are ordered in decreasing order of the relative cost differences. There are 19 households that obtain a reduction higher than 10%, 30 households that obtain a reduction higher than 5% and only 7 households that do not obtain any reduction.

In case of net purchase and sale mechanism, the cost savings by month are shown in Table III. The annual cost for the community if each household pay by her own net consumption is $55 609.93, corresponding to a monthly average per household of $57.93. If the residents share their solar rooftop generation and allocate the costs according to Allocation 2 (18)–(19), the total annual cost is $42 973.74 corresponding to a monthly average cost per household of $44.76 and a relative cost reduction of 22.72%.

The distribution of the monthly costs and savings are depicted in Figures 7 and 8. As was analytically explained in Section IV-D, in this case the cost savings are much higher than for the net metering billing mechanism. Moreover, unlike

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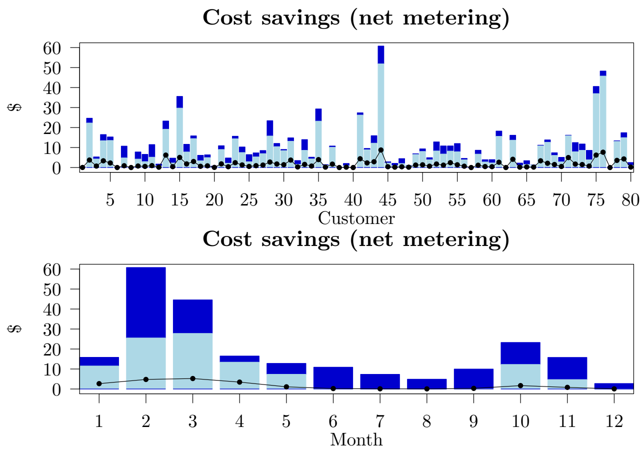

Fig. 6. Community savings for net metering billing mechanism

TABLE IV
SUMMARY OF COST SAVINGS FOR NET METERING (II)

Prosumer	(a)	(b)	(c)	(d)
5218	−3.84	−38.55	34.71	903.19
8645	−2.46	−20.54	18.07	733.38
2199	−4.48	−26.48	22.00	490.63
9937	10.15	−32.74	42.90	422.42
171	11.90	−28.52	40.42	339.60
3456	−20.98	−65.78	44.81	213.59
8995	−41.87	−101.66	59.79	142.80
8243	45.88	5.43	40.45	88.16
7024	34.27	5.24	29.02	84.70
8046	−86.57	−136.06	49.50	57.18
9971	137.10	85.52	51.59	37.63
5129	91.92	64.14	27.78	30.22
1800	129.89	93.65	36.24	27.90
9001	132.07	111.33	20.73	15.70
3538	122.99	104.27	18.73	15.23
5874	108.81	93.97	14.84	13.63
1792	168.54	146.88	21.66	12.85
1283	670.01	595.46	74.55	11.13
6063	209.11	188.03	21.08	10.08
2945	120.44	109.41	11.03	9.16

(a) Cost without sharing ($), (b) Cost with sharing ($)
(c) Cost savings (a)–(b) ($), (d) Cost savings (c)/|(a)| (%)

that case, the higher savings are not concentrated in two months and for a small number of prosumers.

In Table VI, we show the savings for the households. Similar to the net metering case, we only show the 20 prosumers that obtain higher annual savings and they are ordered in decreasing order of the relative cost savings. Every household obtains a significant reduction of her energy cost in this case. There are 21 households that obtain a reduction higher than 50%, 52 households obtain a reduction higher than 20%, 70 higher than 10% and only one has a reduction lower than 1%.

We could also validate our comparative analysis between the mechanisms. Net metering produces lower costs for the households than net purchase and sale. But from the point of view of saving cost due to sharing, net purchase and sale is the most interesting as it promotes association by an effective sharing of the energy excesses and producing significant reduction of the energy costs for every household.

TABLE V
SUMMARY OF COST SAVINGS FOR NET PURCHASE AND SALE

Month	(a)	(b)	(c)	(d)
1	2773.90	1570.02	1203.88	43.40
2	1336.72	136.45	1200.27	89.79
3	1694.22	404.29	1289.93	76.14
4	2500.15	1253.37	1246.78	49.87
5	4378.49	3289.97	1088.52	24.86
6	6802.61	5765.62	1036.99	15.24
7	8442.62	7444.56	998.06	11.82
8	8077.89	7209.74	868.15	10.75
9	7071.99	6106.10	965.89	13.66
10	4572.32	3358.26	1214.06	26.55
11	3353.83	2445.12	908.72	27.09
12	4605.20	3990.26	614.95	13.35
Total	55 609.93	42 973.74	12 636.18	22.72

(a) Cost without sharing ($) (b) Cost with sharing ($)
(c) Cost savings (a)–(b) ($) (d) Cost savings (c)/|(a)| (%)

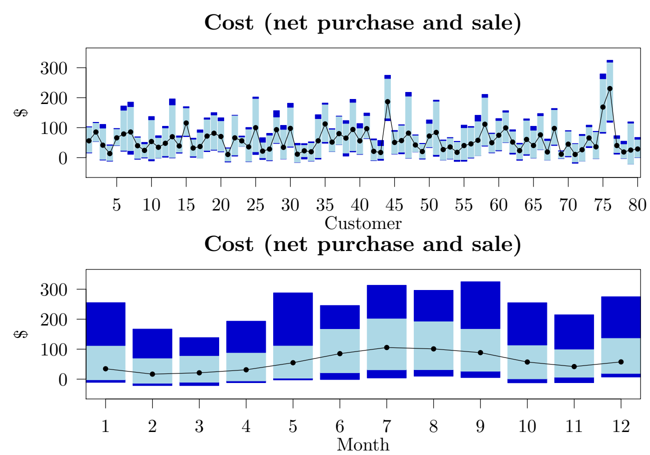

Fig. 7. Community cost for net purchase and sale billing mechanism

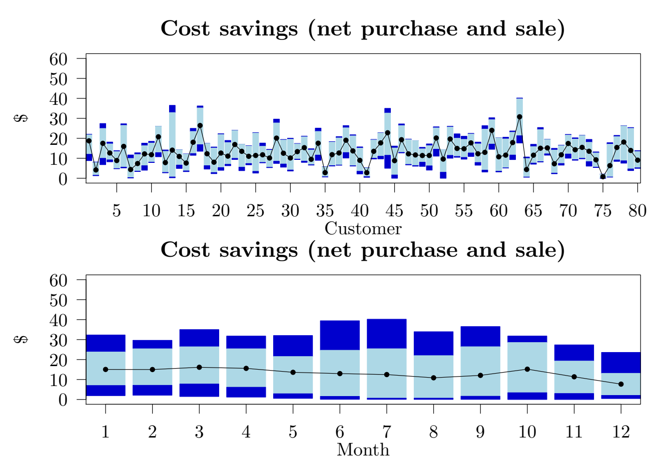

Fig. 8. Community savings for net purchase and sale billing mechanism

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TABLE VI
SUMMARY OF COST SAVINGS FOR NET PURCHASE AND SALE (II)

Prosumer	(a)	(b)	(c)	(d)
8995	129.49	−41.87	171.37	132.34
8046	282.91	−86.57	369.47	130.60
3456	139.16	−20.98	160.14	115.07
2199	128.52	−4.48	133.00	103.49
5218	208.66	−3.84	212.50	101.84
8645	138.64	−2.46	141.11	101.78
9937	227.65	10.15	217.50	95.54
171	163.75	11.90	151.85	92.73
7024	213.38	34.27	179.12	83.94
8243	228.18	45.88	182.30	79.89
1800	446.96	129.89	317.07	70.94
3482	276.09	92.10	183.99	66.64
5129	253.90	91.92	161.98	63.80
781	416.68	167.39	249.29	59.83
9001	317.79	132.07	185.72	58.44
1792	385.18	168.54	216.64	56.24
5874	245.45	108.81	136.64	55.67
9971	304.19	137.10	167.08	54.93
6578	429.52	193.67	235.85	54.91
2945	261.49	120.44	141.05	53.94

(a) Cost without sharing ($) (b) Cost with sharing ($)
(c) Cost savings (a)–(b) ($) (d) Cost savings (c)/|(a)| (%)

It is interesting to analyze the prosumers that obtain greater benefits by aggregation under the net purchase and sale program. Their monthly net generation G_i^{NPS} and net consumption Q_i^{NPS} are reflected in Tables VII and VIII. The prosumers obtain aggregation benefits under net metering in those months when their individual net consumptions have opposite signs with respect to that of the group. Moreover except for prosumer 8046, all the others obtain relatively high benefits under net metering program in those months, but the benefits are concentrated only in a few months. There are no benefits by aggregation under net metering program on those months where $D_i D_{\mathcal{N}} \geq 0$ for all i. This happens in January, February, March, April and August. However, we can see in Tables VII and VIII that neither G_i^{NPS} nor Q_i^{NPS} are zero in those months. This means, that there are time instants where the condition $D_i(t)D_{\mathcal{N}}(t) < 0$ is satisfied and the net purchase and sale program still provides benefits by aggregation. For this reason, net purchase and sale mechanism provides more quantitative benefits under aggregation annually.

VI. CONCLUSIONS

Drastic cost reduction in the PV systems technology in the last few years has resulted in significant increase in their worldwide installation. Attractive billing methods implemented by different system operators have also encouraged more houses to install PV systems. In this paper, we have explored the idea of sharing the electricity generation by PV systems of different households among each other and improving their profits promoting the use of clean energy more in the power system. We considered sharing under three different programs: feed-in tariff, net metering, and net purchase and sale. In feed-in tariff, there is no advantage in sharing. In net metering, and net purchase and sale, sharing is advantageous if and only if the retail price of electricity by the utility is more than the price of selling electricity to the utility. Under that favorable sharing condition, we found rules for allocating their joint cost based on cost causation principle. The allocations also follow standalone cost principle – *i.e.* they are in the core of the cooperative games of net electricity consumption cost. We have also performed a comparative analysis between net metering, and net purchase and sale. We have verified our developed results in the data set of a community of residential households in Austin, Texas. Our results show that net purchase and sale mechanism promotes aggregation of solar energy better than net metering in scenarios where most of the prosumers have similar net consumption patterns. The reason is that net purchase and sale accounts for instantaneous net consumption instead of overall net consumption while calculating savings in a billing period. These results will definitely attract more households to share their PV systems generated electricity with each other and as a result, the whole society will benefit. We think that our results are of prime interest for regulators and policy makers because the results provide scientific evidence to design tariffs and billing mechanisms that promote cooperation and make more efficient use of the renewable energy at the local level. We are currently extending our results to scenarios where the prosumers also

TABLE VII
NET GENERATION G_i^{NPS} FOR EACH MONTH

Month	8995	8046	3456	2199
1	212.95	681.59	246.97	177.74
2	631.88	1039.38	606.38	463.40
3	598.41	1010.57	574.65	425.61
4	302.00	263.50	258.00	242.00
5	477.74	796.83	427.43	320.73
6	427.56	833.84	359.35	402.46
7	402.24	855.14	347.76	359.90
8	357.63	716.21	327.96	271.68
9	399.07	816.66	347.29	229.39
10	558.56	946.18	477.05	263.05
11	416.84	596.44	380.73	193.96
12	285.38	373.03	236.80	159.91
Total	5070.26	8929.37	4590.37	3509.83

All data are given in kWh

TABLE VIII
NET CONSUMPTION Q_i^{NPS} FOR EACH MONTH

Month	8995	8046	3456	2199
1	544.71	913.26	495.13	407.90
2	190.29	511.82	200.47	132.50
3	247.20	565.91	200.57	163.66
4	418.00	456.50	462.00	478.00
5	332.93	656.93	274.20	252.30
6	559.41	832.13	476.53	399.75
7	652.68	848.30	634.08	504.39
8	626.96	846.18	533.95	439.53
9	538.62	758.60	500.80	399.62
10	386.80	605.15	367.50	304.56
11	202.67	577.14	259.78	188.97
12	227.48	607.07	266.99	197.68
Total	4927.75	8178.99	4672.00	3868.86

All data are given in kWh

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have storage systems. They could share solar PV as well as storage system within the community, and obtain more benefit taking advantage of time-of-use tariffs.

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APPENDIX

A. Proof of Theorem 2

Let us consider two disjoint coalitions $\mathcal{S}$ and $\mathcal{T}$, then

$$C^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} = \lambda Q^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} - \mu G^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}},$$

where

$$\begin{aligned} Q^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} &= (Q^{\text{NM}}_{\mathcal{S}} + Q^{\text{NM}}_{\mathcal{T}} - G^{\text{NM}}_{\mathcal{S}} - G^{\text{NM}}_{\mathcal{T}})_+, \\ G^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} &= (G^{\text{NM}}_{\mathcal{S}} + G^{\text{NM}}_{\mathcal{T}} - Q^{\text{NM}}_{\mathcal{S}} - Q^{\text{NM}}_{\mathcal{T}})_+, \end{aligned}$$

and

$$C^{\text{NM}}_{\mathcal{S}} + C^{\text{NM}}_{\mathcal{T}} = \lambda(Q^{\text{NM}}_{\mathcal{S}} + Q^{\text{NM}}_{\mathcal{T}}) - \mu(G^{\text{NM}}_{\mathcal{S}} + G^{\text{NM}}_{\mathcal{T}}).$$

In net metering mechanism $Q^{\text{NM}}_{\mathcal{S}} \geq 0$, $G^{\text{NM}}_{\mathcal{S}} \geq 0$, and one of them is always zero, – *i.e.* $Q^{\text{NM}}_{\mathcal{S}} G^{\text{NM}}_{\mathcal{S}} = 0$ for any coalition $\mathcal{S} \subseteq \mathcal{N}$. Then we can distinguish four possible cases:

Case (I): $G^{\text{NM}}_{\mathcal{S}} = G^{\text{NM}}_{\mathcal{T}} = 0$, then

$$\begin{aligned} Q^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} &= (Q^{\text{NM}}_{\mathcal{S}} + Q^{\text{NM}}_{\mathcal{T}})_+ = Q^{\text{NM}}_{\mathcal{S}} + Q^{\text{NM}}_{\mathcal{T}}, \\ G^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} &= (-Q^{\text{NM}}_{\mathcal{S}} - Q^{\text{NM}}_{\mathcal{T}})_+ = 0, \end{aligned}$$

and

$$C^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} = \lambda(Q^{\text{NM}}_{\mathcal{S}} + Q^{\text{NM}}_{\mathcal{T}}) = C^{\text{NM}}_{\mathcal{S}} + C^{\text{NM}}_{\mathcal{T}}.$$

Case (II): $Q^{\text{NM}}_{\mathcal{S}} = Q^{\text{NM}}_{\mathcal{T}} = 0$, then

$$\begin{aligned} Q^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} &= (-G^{\text{NM}}_{\mathcal{S}} - G^{\text{NM}}_{\mathcal{T}})_+ = 0, \\ G^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} &= (Q^{\text{NM}}_{\mathcal{S}} + Q^{\text{NM}}_{\mathcal{T}})_+ = Q^{\text{NM}}_{\mathcal{S}} + Q^{\text{NM}}_{\mathcal{T}}, \end{aligned}$$

and

$$C^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} = -\mu(G^{\text{NM}}_{\mathcal{S}} + G^{\text{NM}}_{\mathcal{T}}) = C^{\text{NM}}_{\mathcal{S}} + C^{\text{NM}}_{\mathcal{T}}.$$

Case (III): $G^{\text{NM}}_{\mathcal{S}} = Q^{\text{NM}}_{\mathcal{T}} = 0$,

$$\begin{aligned} C^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} &= \lambda(Q^{\text{NM}}_{\mathcal{S}} - G^{\text{NM}}_{\mathcal{T}})_+ - \mu(G^{\text{NM}}_{\mathcal{T}} - Q^{\text{NM}}_{\mathcal{S}})_+, \\ C^{\text{NM}}_{\mathcal{S}} + C^{\text{NM}}_{\mathcal{T}} &= \lambda Q^{\text{NM}}_{\mathcal{S}} - \mu G^{\text{NM}}_{\mathcal{T}}. \end{aligned}$$

We have two cases, either $Q^{\text{NM}}_{\mathcal{S}} \geq G^{\text{NM}}_{\mathcal{T}}$ or $Q^{\text{NM}}_{\mathcal{S}} < G^{\text{NM}}_{\mathcal{T}}$. In both cases, $C^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} \leq C^{\text{NM}}_{\mathcal{S}} + C^{\text{NM}}_{\mathcal{T}}$ if and only if $\mu \leq \lambda$.

Case (IV): $Q^{\text{NM}}_{\mathcal{S}} = G^{\text{NM}}_{\mathcal{T}} = 0$,

$$\begin{aligned} C^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} &= \lambda(Q^{\text{NM}}_{\mathcal{T}} - G^{\text{NM}}_{\mathcal{S}})_+ - \mu(G^{\text{NM}}_{\mathcal{S}} - Q^{\text{NM}}_{\mathcal{T}})_+, \\ C^{\text{NM}}_{\mathcal{S}} + C^{\text{NM}}_{\mathcal{T}} &= \lambda Q^{\text{NM}}_{\mathcal{T}} - \mu G^{\text{NM}}_{\mathcal{S}}. \end{aligned}$$

We have two cases, either $Q^{\text{NM}}_{\mathcal{T}} \geq G^{\text{NM}}_{\mathcal{S}}$ or $Q^{\text{NM}}_{\mathcal{T}} < G^{\text{NM}}_{\mathcal{S}}$. In both cases, $C^{\text{NM}}_{\mathcal{S}\cup\mathcal{T}} \leq C^{\text{NM}}_{\mathcal{S}} + C^{\text{NM}}_{\mathcal{T}}$ if and only if $\mu \leq \lambda$.

Thus, subadditivity of the cost function C^{NM} of the cooperative game $(\mathcal{N}, C^{\text{NM}})$ is equivalent to $\lambda \geq \mu$.

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B. Proof of Theorem 3

According to Definition 9, we have to prove that the cost allocation rule given by (13) satisfies the Axioms 1–4 and 6–7. Let $D_i = Q_i^{\text{NM}} - G_i^{\text{NM}}$ be the net consumption. The allocation follows the following axioms:

Axiom 1 (Equity): It is easy to see that if two households i and j have same net consumptions, the allocated costs must be the same – *i.e.* if $D_i = D_j$ then $x_i^{\text{NM}} = x_j^{\text{NM}}$.

Axiom 2 (Monotonicity): Under the cost allocation rule (13), if $D_i D_j \geq 0$ and $|D_i| \geq |D_j|$ then $|x_i^{\text{NM}}| \geq |x_j^{\text{NM}}|$ and this proves Monotonicity.

Axiom 3 (Individual Rationality):

$$C_i^{\text{NM}} = \begin{cases} \lambda Q_i^{\text{NM}}, & \text{if } D_i \geq 0, \\ -\mu G_i^{\text{NM}}, & \text{if } D_i < 0. \end{cases} \tag{25}$$

Since $\lambda \geq \mu$, comparing (13) with (25) we can say that the allocated cost will be less than the net consumption cost if the household would not have joined the aggregation – *i.e.* $x_i^{\text{NM}} \leq C_i^{\text{NM}}$.

Axiom 4 (Budget Balance):

If $D_{\mathcal{N}} \geq 0$:

$$\sum_{i\in\mathcal{N}} x_i^{\text{NM}} = +\sum_{i\in\mathcal{N}} \lambda(Q_i^{\text{NM}} - G_i^{\text{NM}}) = C_{\mathcal{N}}^{\text{NM}},$$

If $D_{\mathcal{N}} < 0$:

$$\sum_{i\in\mathcal{N}} x_i^{\text{NM}} = -\sum_{i\in\mathcal{N}} \mu(G_i^{\text{NM}} - Q_i^{\text{NM}}) = C_{\mathcal{N}}^{\text{NM}}.$$

So the cost allocation rule is such that the sum of allocated costs are equal to the total net electricity consumption cost – *i.e.*

$$\sum_{i\in\mathcal{N}} x_i^{\text{NM}} = C_{\mathcal{N}}^{\text{NM}}.$$

Recall from Definition 8 that a household i with positive net consumption $D_i = Q_i^{\text{NM}}$ is causing cost to the system and with negative $D_i = -G_i^{\text{NM}}$ is mitigating cost to the system.

Axiom 6 (Penalty for causing cost): From (13), if $D_i = Q_i^{\text{NM}} \geq 0$, $x_i^{\text{NM}} \geq 0$ – *i.e.* those individuals causing cost will pay for it.

Axiom 7 (Reward for cost mitigation): From (13), if $D_i = -G_i^{\text{NM}} < 0$, $x_i^{\text{NM}} < 0$ – *i.e.* those individuals mitigating cost will be rewarded. The rate of penalty and reward is same here.

And we conclude that the cost allocation defined by (13) is a cost causation based allocation.

C. Proof of Theorem 4

In order to prove that the cost allocation rule satisfies the standalone principle (Axiom 5), two cases are considered:

If $D_{\mathcal{N}} \geq 0$:

$$\sum_{i\in S} x_i^{\text{NM}} = \lambda D_{\mathcal{S}},$$
$$C_{\mathcal{S}}^{\text{NM}} = +\lambda Q_{\mathcal{S}}^{\text{NM}}, \quad \text{if } D_{\mathcal{S}} \geq 0,$$
$$C_{\mathcal{S}}^{\text{NM}} = -\mu G_{\mathcal{S}}^{\text{NM}}, \quad \text{if } D_{\mathcal{S}} \leq 0.$$

If $D_{\mathcal{N}} \leq 0$:

$$\sum_{i\in S} x_i^{\text{NM}} = \mu D_{\mathcal{S}},$$
$$C_{\mathcal{S}}^{\text{NM}} = +\lambda Q_{\mathcal{S}}^{\text{NM}}, \quad \text{if } D_{\mathcal{S}} \geq 0,$$
$$C_{\mathcal{S}}^{\text{NM}} = -\mu G_{\mathcal{S}}^{\text{NM}}, \quad \text{if } D_{\mathcal{S}} \leq 0.$$

So from above, we can conclude that for every aggregation $\mathcal{S} \subseteq \mathcal{N}$:

$$\sum_{i\in S} x_i^{\text{NM}} \leq C_{\mathcal{S}}^{\text{NM}},$$

and this proves that the cost allocation satisfies the standalone cost principle.

D. Proof of Theorem 5

The cooperative game $(\mathcal{N}, C^{\text{NPS}})$ for the billing period $[t_0, t_f]$ is subadditive if and only if the game $(\mathcal{N}, C^{\text{NPS}}(t))$ for $t \in [t_0, t_f]$ is subadditive. Since the instantaneous game $(\mathcal{N}, C^{\text{NPS}}(t))$ is equivalent to a net metering coalitional game for a billing period reduced to the time instant t, then by Theorem 2, subadditivity is equivalent to $\lambda \geq \mu$.

E. Proof of Theorem 6

The cost allocation $x_i^{\text{NPS}}(t)$ given by (19) satisfies Axioms 1–7 for the instantaneous cooperative game $(\mathcal{N}, C^{\text{NPS}}(t))$ where $t \in [t_0, t_f]$ because the game $(\mathcal{N}, C^{\text{NPS}}(t))$ is equivalent to a net metering cooperative game where the billing interval reduces to the time instant t. Thus, $x_i^{\text{NPS}} = \int_{t_0}^{t_f} x_i^{\text{NPS}}(t)dt$ is a cost causation based cost allocation that belongs to the core of the cooperative game $(\mathcal{N}, C^{\text{NPS}})$ for the net purchase and sale program.

F. Proof of Theorem 7

We consider two cases.

Case $D_{\mathcal{S}} \geq 0$: There is nonnegative net consumption, then $D_{\mathcal{S}} = Q_{\mathcal{S}}^{\text{NM}} = Q_{\mathcal{S}}^{\text{NPS}} - G_{\mathcal{S}}^{\text{NPS}}$. The cost of the coalition consumption under Net metering and net purchase and sale programs are:

$$C_{\mathcal{S}}^{\text{NM}} = \lambda D_{\mathcal{S}} = \lambda Q_{\mathcal{S}}^{\text{NPS}} - \lambda G_{\mathcal{S}}^{\text{NPS}},$$
$$C_{\mathcal{S}}^{\text{NPS}} = \lambda Q_{\mathcal{S}}^{\text{NPS}} - \mu G_{\mathcal{S}}^{\text{NPS}}.$$

Thus,

$$C_{\mathcal{S}}^{\text{NPS}} - C_{\mathcal{S}}^{\text{NM}} = (\lambda - \mu) G_{\mathcal{S}}^{\text{NPS}}.$$

Case $D_{\mathcal{S}} < 0$: There is positive net generation, then $D_{\mathcal{S}} = -G_{\mathcal{S}}^{\text{NM}} = Q_{\mathcal{S}}^{\text{NPS}} - G_{\mathcal{S}}^{\text{NPS}}$. The cost of the coalition consumption under net metering and net purchase and Sale programs are:

$$C_{\mathcal{S}}^{\text{NM}} = \mu D_{\mathcal{S}} = \mu Q_{\mathcal{S}}^{\text{NPS}} - \mu G_{\mathcal{S}}^{\text{NPS}},$$
$$C_{\mathcal{S}}^{\text{NPS}} = \lambda Q_{\mathcal{S}}^{\text{NPS}} - \mu G_{\mathcal{S}}^{\text{NPS}}.$$

Thus,

$$C_{\mathcal{S}}^{\text{NPS}} - C_{\mathcal{S}}^{\text{NM}} = (\lambda - \mu) Q_{\mathcal{S}}^{\text{NPS}}.$$