Cost Causation Based Allocations of Costs for Market Integration of Renewable Energy

Pratyush Chakraborty, *Member, IEEE*, Enrique Baeyens, *Member, IEEE*, and Pramod P. Khargonekar, *Fellow, IEEE*

***Abstract*—An important problem in modern electrical power systems is the inherent variability and uncertainty of renewable energy resources. Allocating the costs generated by the variable resources in a just and reasonable manner is crucial for economic efficiency. In this paper, we develop an axiomatic framework for allocating the deviation costs generated by variable resources using the cost causation based principle. The cost causation principle has a long historical tradition in electrical energy systems and has been proved to provide economic efficiency. We apply the new cost causation based framework to allocate the cost in the production deviation of a group of renewable energy producers and thereby help promote the market integration of renewable energy. The resulting cost allocation framework is then applied to five wind power producers bidding in the Iberian electricity market in Europe, using real data on wind speeds and market prices.**

***Index Terms*—Cost allocation, cost causation principle, cooperative games, renewable integration.**

NOMENCLATURE

Subindices and superindices

t	Subindex of time.
i, j, k	Superindices of REPs.
$\mathcal{S}, \mathcal{N}$	Superindices of sets, also used as coalitions.

Constants and variables

q, λ	Deviation penalties.
$\mathbf{d}_t$	Vector of deviations at time t.
d_t^i	Deviation of REP i at time t.
$d_t^{\mathcal{S}}, d_t^{\mathcal{N}}$	Net deviation of coalition $\mathcal{S}, \mathcal{N}$.
$\mathbf{x}$	Allocation vector.
$\mathbf{w}_t$	Vector of realized renewable power at time t.
w_t^i	Realized renewable power of REP i at time t.
$\mathbf{C}_t$	Vector of contracts in the day ahead market at time t.
C_t^i	Contract of REP i in the day ahead market at time t.
$\mathbf{m}$	Vector of cost cause values.
m^i	Value of cost cause i.
$m^{\mathcal{S}}$	Net value of cost causes in set $\mathcal{S}$.

Sets

$\mathcal{N}$	Set of all REPs, grand coalition.
$2^{\mathcal{N}}$	Power set of the grand coalition.
$\mathcal{S}$	Coalition, a subset of the grand coalition.
$\mathcal{C}$	The core, the set of all stabilizing allocations.
$\mathcal{P}_t, \mathcal{Q}_t$	Set of REPs with nonnegative and negative deviation, respectively.
Γ, Γ_i	Smooth paths.

Functions

θ, θ_c	Cost functions.
v	Value function.
e	Excess function.
π_t^i	Allocation rule.
α	Balanced map.
$\mathbf{1}_{\mathcal{S}}$	Indicator function of set $\mathcal{S}$.
$(\cdot)_+$	Positive part function.

Operators

∇	Gradient or derivative operator.
∇^i	Partial derivative with respect to the ith variable.
$\vert \cdot \vert$	Absolute value or set cardinality.
$\setminus$	Set difference.
$\circ$	Path concatenation.

Manuscript received April 9, 2016; revised July 25, 2016, November 23, 2016, and March 12, 2017; accepted March 23, 2017. Date of publication March 31, 2017; date of current version December 20, 2017. This work was supported in part by the National Science Foundation under Grant ECCS-1129061 and Grant CNS-1239274. Paper no. TPWRS-00556-2016. *(Corresponding author: Pratyush Chakraborty.)*

P. Chakraborty is with the Department of Mechanical Engineering, University of California, Berkeley, CA 94720 USA (e-mail: prats19@gmail.com).

E. Baeyens is with the Instituto de las Tecnologías Avanzadas de la Producción, Universidad de Valladolid, 47002 Valladolid, Spain (e-mail: enrbae@eis.uva.es).

P. P. Khargonekar is with the University of California, Irvine, CA 92697 USA (e-mail: ppk@ufl.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TPWRS.2017.2690404

I. INTRODUCTION

THERE is great interest in large-scale integration of renewable resources such as wind and solar into the existing power grid in many parts of the world. Carbon emissions and pollution, climate change, and sustainability are the key driving forces motivating this transformation of the electric energy supply. But unlike traditional power sources, these renewable sources are inherently uncertain, variable and non-dispatchable [1]–[3]. Technical challenges posed by the need to balance the demand with supply at all times are foreseen as a major concern in power system operations with deep renewable penetration [4]–[6]. Significant deviations between the day-ahead or intra-day offered contract value and the delivered power in real time are expected to occur. Since grid-scale storage is not yet

0885-8950 © 2017 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.
See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

widespread and sufficient, due largely to the high cost of the storage devices, the system operator needs to use the ancillary services to ensure supply-demand balance and the incurred costs have to be allocated to those that are responsible for the deviations.

Wind and solar power installations have increased significantly in recent years. In 2015, wind and solar were respectively first and third in newly installed capacity additions in the U.S. according to preliminary data from the U.S. Energy Information Administration [7]. Traditionally, these renewable energy sources have been subsidized because of their potential to contribute to decarbonization of the energy sector. For example, they are exempt from paying for the increases in ancillary services caused by their inherent uncertainty and variability; see for example [8], where a survey of fifteen European countries on wind generation imbalance penalties is included. In half of the surveyed countries, the wind energy is not penalized for imbalances. In the U.S., the Federal Energy Regulatory Commission (FERC) under Order 890, established the imbalance settlement, with different conditions for conventional and intermittent resources. The specific case of each market in North America is summarized in the report [9]. As wind and solar penetration increases, there is likely to be growing demand for allocating the increased integration and management costs to those causing the increase in a rational manner.

Aggregation of geographically diverse wind power plants has considerable potential for reducing the variability of wind generation and lessening its integration costs. The underlying idea is to exploit the low correlation that may exist among the renewable sources [10]. Aggregation is a very powerful approach that has been traditionally used in electrical power systems to increase system reliability and reduce costs. It provides benefits because many individual requirements (contingency reserves, peak load, regulation, etc.) are not completely correlated. For example, the aggregated system load is much more predictable than the individual loads. In the same way, the aggregated renewable energy supply is less variable than the supply of an individual renewable energy producer. The contingency reserves provide another example in which many generators can share the same reserve pool. In all such cases, the issue of allocating total incurred costs at the aggregate level to individual members of the aggregation becomes an important problem in power systems.

We propose here to use the cost causation principle to develop efficient mechanisms to allocate the aggregated cost among the individual members that generated them. The main reason to use the cost causation principle is its long tradition in the U.S. electric markets. In the United States, the principal statute that governs electric-power transactions is the Federal Power Act (FPA). Under the FPA, FERC can only approve a *just and reasonable* tariff [11]. The U.S. courts have defined the concept *just and reasonable* by developing the cost causation principle. Under that principle, FERC tariff approvals must demonstrate that the parties bearing facility costs receive benefits that are *roughly commensurate* [11]. It is now widely recognized that the retail rates based on the cost causation principle help the economy to be more efficient and competitive [11], [12]. The cost causation principle implies that costs should be borne by those who cause them to be incurred. By correctly calculating and allocating cost impacts, transparent signals are sent to markets and regulators that can encourage economic efficiency, and avoid subsidies [12], [13].

The locational marginal price (LMP) is a good illustration of the cost causation principle. The total cost to meet load and losses is the summation of all the generation and transmission costs. Though the net load causes the total cost, the price implied is not the average of the costs over the load. At a given location, LMP is derived using marginal effects of generation, transmission and losses needed to meet incremental load. In a sufficiently well-behaved system, this LMP is well defined and provides the right incentives for generation and load [12].

Applying the cost causation principle to the costs supporting renewable capacity will reward appropriate customer behavior needed to reduce system costs and promote competition among industrial ratepayers, thus in turn making renewable integration less burdensome.

Starting from the Bonbright's principles for rate making [14], Kirby *et al.* [15] established a set of rules for defining a tariff based on the cost-causation principle to be applied to the cost of ancillary services. Their rules are declarative and have not been expressed in mathematical form. They also do not consider the problem of sharing the imbalance cost among the various users that incurred it. Motivated by these rules, we formulate an axiomatic framework for cost causation based allocations. This is a completely general framework that can be applied to many different problems in electrical power systems related to the integration of variable resources either on the supply or on the demand side. Our goal is to define an axiomatic framework that incorporates the declarative rules [15] and develop a general mechanism for allocating the joint costs of imbalance among a group of agents that cooperate and aggregate to reduce and share the incurred costs.

We apply our axiomatic formulation of cost causation based allocations to the market integration of renewable energy resources by considering a group of renewable energy producers (REP) bidding in a competitive two-settlement market system consisting of an *ex-ante* forward market and an *ex-post* imbalance mechanism to penalize uninstructed deviations. Each REP computes the power contract based on the day-ahead electricity price and the expected values of shortfall and surplus penalties. In this model, a payment sharing mechanism to allocate the expected profit of an aggregation of REPs has been developed in [16], [17]. This mechanism has two weaknesses that prevent its full adoption in practice. First, the REPs need to agree on a common statistical model of the renewable resource to compute the joint power contract. And second, they have to share the realized profit, not the expected profit. Coalitions can break either because they need to communicate and agree on common statistical models on their individual proprietary resources or because the realized profit is far from the expected profit during various time periods and some of them are dissatisfied during these periods with an allocation mechanism that is based on the expected values. To avoid these drawbacks, we extend our previous results to the case where the REPs do not necessarily share any statistical information with each other and the aggregator bids a contract that has been obtained as the sum of the individual contracts, independent of how each REP computes her own individual contract. At the time of delivery, the produced power can be different from the contract. The cost of this net deviation has to be allocated among the individual REPs in a *just and reasonable* way.

Our main motivation for developing an axiomatic formulation of cost causation based allocations is the integration of renewable resources, however the framework can be applied to other problems in electrical systems. Beyond renewable resources, there are other types of generators that may have difficulty in following a set-point signal and are causing the need of additional balance. There are also implications for non-conforming loads, such as arc-furnaces or rolling mills that introduces erratic, large magnitude, and short duration load changes. Another interesting application that can benefit for a cost causation based allocation framework is demand response where a large number of customers enroll in a program to jointly reduce the net consumption and obtain an economic profit that should be allocated among the participants. There can be different types of problem formulations in demand response and it will be interesting to extend our framework to those cases. A cost causation based allocation could increase wider adoption of such schemes. Besides, by penalizing those that produce imbalance and rewarding those that mitigate it, transparent signals are sent that promote economic efficiency and avoid subsidies.

The problem of cost allocation has been extensively studied in the literature. Different allocation methods under the setting we consider have been proposed in [18]–[20]. There also exist cost sharing rules based on general principles or axioms [21]–[23] satisfying certain properties, but none of these mechanisms have been previously analyzed under the framework of the cost causation principle.

The main contributions of this paper can be summarized as follows: We develop a general axiomatic formulation based on the cost causation principle for sharing the deviation costs incurred by a group of agents that decide to cooperate to reduce this deviation. We apply this framework to the electric-power systems with renewables where the cost is incurred by uninstructed deviations in generation. We obtain in this case two allocations that can be expressed in analytical form. The sharing of cost problem can also be formulated as a cooperative game and we prove that one of the developed allocations, the cost causation based allocation with nonzero reward, lies in the core of the game. This allocation can also be connected to classical general allocation rules with multiple cost causes like the Aumann-Shapley and the Friedman-Moulin rules using an *a posteriori* linear cost function. The cost causation based allocation with nonzero reward is very appealing because, contrary to some popular allocations such as the Shapley value or the nucleolus [24], [25], it is straightforward to compute for a very large number of players because its computational complexity does not increase exponentially with the number of agents. To the best of our knowledge, there is no other axiomatic formulation for sharing aggregated costs based on the cost causation principle in the literature and the general cost allocation rules have not previously analyzed under this principle.

Another important contribution is an empirical case study using real data on wind speeds and prices for five wind farms located in the Iberian peninsula. In this study, we show that if the five wind farms decide to share the net deviation, they are always satisfied with the cost allocation given by the the cost causation based allocation with nonzero reward. The allocation is not very different from that given by the Shapley-Shubik rule. However, even though they are not very different in the allocated quantites, the latter fails in about half of the analyzed cases to satisfy all the wind farms.

The remainder of this paper is organized as follows. In Section II, we formally define a general deviation cost allocation problem and develop an axiomatic framework for characterizing allocations that follow the cost causation principle. In Section III, we derive two new cost causation based allocations using the axiomatic approach and prove that one of them lies in the core of the corresponding cooperative game. In Section IV, we analyze the general cost allocation rules in our cost causation based axiomatic framework. Section V presents a comparative example of all the existing allocation rules and newly developed rules in this paper and also includes an empirical case study based on real data. Finally, we present some conclusions in Section VI.

II. Cost Causation Based Allocation

A. The Cost Allocation Problem

A central planning problem in a company or agency is the *fair* or *just* allocation of common costs. Typical examples are the fees for the use of a common facility such as an airport, a highway, a water reservoir, a communication network or a power grid. This problem has been extensively studied in the economics literature [21]–[23].

We introduce the main elements of a cost allocation problem.

Definition 1 (Set of agents): Let $\mathcal{N} = \{1, 2, \ldots, N\}$ be a set of N agents that share the cost of using a common facility (or the cost of producing a good or service).

Each agent has been assigned an instruction to use a shared facility (or a production order), *e.g.*, by means of a private contract, that is the responsibility of each individual and is not shared with the rest of the agents. Each individual has to pay a penalty if it deviates from the assignment.

Definition 2 (Vector of deviations and net deviation): Let $\mathbf{d}_t = \{d_t^i : i \in \mathbb{N}\} \in \mathbb{R}^N$ be the vector of deviations at time t, where d_t^i is the individual deviation of agent i. The net deviation at time t is given by $d_t^{\mathcal{N}} = \sum_{i \in \mathcal{N}} d_t^i$.

The set of agents $\mathcal{N}$ pay for the cost of net deviation $d_t^{\mathcal{N}}$ according to some cost function θ.

Definition 3 (Cost function): Let $\theta : \mathbb{R} \to \mathbb{R}_+$ be the cost function providing the cost incurred by the net deviation of the set of agents.

The cost function satisfies $\theta(0) = 0$, *i.e.*, there is no cost if the net deviation is zero.

The aggregation of agents provides a benefit if the cost function is subadditive, *i.e.*, if for any $x, y \in \mathbb{R}$, $\theta(x + y) \leq \theta(x) + \theta(y)$. In the sequel, we will consider the following assumption:

Assumption 1 (Subadditivity of the cost function): The cost function θ is subadditive.

The cost allocation problem is to distribute the cost of the net deviation in a *just and reasonable* way among the participating agents. The distribution is given by a cost allocation rule.

Definition 4 (Cost allocation rule): A cost allocation rule is a vector-valued function $\{\pi_t^i(\mathbf{d}_t, \theta) : i \in \mathcal{N}\}$ that distribute the net deviation cost among the set of agents $\mathcal{N}$.

B. Review of Cooperative Game Theory

The cost allocation problem can be formulated as a cooperative game [24], [25]. We present here a brief summary of some of the basic elements of cooperative game theory. Let $2^{\mathcal{N}}$ be the power set of $\mathcal{N}$, *i.e.*, $2^{\mathcal{N}} := \{\mathcal{S} : \mathcal{S} \subseteq \mathcal{N}\}$.

Definition 5 (Coalition and grand coalition): A *coalition* is any subset $\mathcal{S} \in 2^{\mathcal{N}}$ and the *grand coalition* is the set $\mathcal{N}$.

Definition 6 (Cooperative game): A cooperative game is defined by a pair $(\mathcal{N}, v)$, where $v : 2^{\mathcal{N}} \to \mathbb{R}$ is a subadditive value function that assigns a real value to each coalition $\mathcal{S} \subseteq \mathcal{N}$.

A *value allocation* is a vector $\mathbf{x} \in \mathbb{R}^N$, where entry x_i represents the value assigned to player i of the grand coalition.

Definition 7 (Dissatisfaction and excess): The dissatisfaction of a coalition $\mathcal{S}$ with respect to the value allocation x is measured by the excess, that is given by

$$e(\mathbf{x}, \mathcal{S}) = v(\mathcal{S}) - \sum_{i \in \mathcal{S}} x_i. \quad (1)$$

The *worst-case excess* $e^*(\mathbf{x})$ is the maximum value of the excess for any coalition $\mathcal{S} \in 2^{\mathcal{N}}$.

The basic solution concept for a cooperative game is the core that plays a similar role to the Nash equilibrium concept for noncooperative games.

Definition 8 (The core): The core of the cooperative game $(\mathcal{N}, v)$ is defined as follows:

$$\mathcal{C} := \left\{\mathbf{x} \in \mathbb{R}^N : e(\mathbf{x}, \mathcal{N}) = 0,\ e(\mathbf{x}, \mathcal{S}) \geq 0,\ \forall \mathcal{S} \in 2^{\mathcal{N}}\right\}. \quad (2)$$

If a value allocation is in the core, no player has an incentive to break up the grand coalition because the cost does not decrease for any possible subcoalition.

Definition 9 (Balanced game and balanced map): A game $(\mathcal{N}, v)$ is *balanced* if for any balanced map α, $\sum_{\mathcal{S} \in 2^{\mathcal{N}}} \alpha(\mathcal{S}) v(\mathcal{S}) \leq v(\mathcal{N})$ where the map $\alpha : 2^{\mathcal{N}} \to [0, 1]$ is said to be *balanced* if for all $i \in \mathcal{N}$, we have $\sum_{\mathcal{S} \in 2^{\mathcal{N}}} \alpha(\mathcal{S}) \mathbf{1}_{\mathcal{S}}(i) = 1$.

A cooperative game has a nonempty core if and only if it is a balanced game [26], [27]. Balancedness of a game is equivalent to the existence of a nonempty core. Morover, the analytical expression defining a balanced game is a feasible alternative to checking that the core is nonempy. Unfortunately, not every game is balanced.

Consider the cooperative game $(\mathcal{N}, v_t)$ with cost function given by $v_t(\mathcal{S}, \mathbf{d}_t) = \theta(d_t^{\mathcal{S}})$ where $d_t^{\mathcal{S}} = \sum_{i \in \mathcal{S}} d_t^i$ is the net deviation of the coalition $\mathcal{S}$ at time t.

Definition 10 (Stabilizing cost allocation rule): A cost allocation rule is said to be stabilizing for a balanced cooperative game if its range belongs to the core.

A stabilizing cost allocation rule is an appealing candidate for a *just and reasonable* cost allocation.

However, it has several drawbacks. First, not every cooperative game has a nonempty core and second, even if it does, computing an allocation in the core, using the core definition given in equation (2) requires to solve a feasibility program with 2^N constraints that is only tractable when N is small. Thus, we are interested in developing a different approach to calculate *just and reasonable* allocations that could be applied for aggregators or balancing authorities to a very large number of individual agents. Our proposal is to use an axiomatic approach based on the *cost causation principle*. We will develop analytical expressions for two cost allocation rules that are tractable and can be easily computed for any number of players N. Most importantly, one of them is stabilizing. In fact, one of the main contributions in this paper is that the cost causation based axiomatic framework produces a stabilizing allocation rule.

C. The Cost Causation Based Tariffs

In an attempt to develop new mechanisms to mitigate the integration costs of wind energy, Kirby *et al.* [15] foresaw that balancing authorities will move towards tariffs for regulation and imbalance services necessary to integrate wind energy. These tariffs should be based on the cost causation principle by properly allocating the costs to those entities that cause the balancing authority to incur them. The cost-causation based tariffs proposed in [15], [28] are reproduced here:

1) Because maintaining power system reliability is critical, tariffs should base prices on costs so that the costs of maintaining reliability are obvious.
2) Tariffs should be based on cost-causation and the cost of providing the service.
    a) Those individuals who cause costs to the system should pay for those costs.
    b) Those individuals who mitigate costs to the system should either incur a lower cost or be paid for helpful actions.
    c) Complex systems like electric grids produce both joint products and joint costs of production that must be allocated among users of the system.
    d) Tariffs should allocate joint production costs on the basis of the use of joint products.
3) Tariffs should not collect revenue if no cost is incurred.
4) Tariffs should be based on the physical behavior and characteristics of the power system.
    a) Recognize the need to balance aggregate system load and aggregate system generation.
    b) Recognize that balancing individual loads or resources is unnecessary and inconsistent with power system operations.
5) Tariffs should result in an efficient allocation of resources.
6) Tariffs should also support the broader principles of horizontal and vertical consistency. Horizontal consistency means that if two individuals cause equal increases in costs, then the tariff should assess each the same amount. Vertical consistency implies that if an individual imposes a larger cost, they should pay more. Both principles can be extended to cost mitigation.

D. An Axiomatic Formulation of Allocations Based on the Cost Causation Principle

The tariffs proposed in [15] for integration of variable generation have been our main inspiration to develop an axiomatic formulation for cost allocations based on the cost causation principle. These cost causation based tariffs were proposed as a declaration of principles, but they lack a rigorous mathematical formulation. In addition, no cost allocation mechanisms have been derived from them.

We propose here five axioms that characterize *just and reasonable* allocation rules. These axioms are: *equity*, *monotonicity*, *individual rationality*, *budget balance*, and *standalone cost principle*. These general principles can be formally established as follows:

Axiom 1 (Equity): If two agents i and j have same deviations, the allocated deviation costs must be the same *i.e.*, if $d_t^i = d_t^j$ then $\pi_t^i(\mathbf{d}_t) = \pi_t^j(\mathbf{d}_t)$.

Axiom 2 (Monotonicity): If two agents i and j have deviations of the same sign, and agent i has a higher deviation than agent j, then the absolute value of the allocated cost to i must be higher than the absolute value of the allocated cost to j *i.e.*, if $d_t^i d_t^j \geq 0$ and $|d_t^i| \geq |d_t^j|$ then $|\pi_t^i(\mathbf{d}_t)| \geq |\pi_t^j(\mathbf{d}_t)|$.

Axiom 3 (Individual Rationality): The allocated cost must be less than the deviation cost if the agent would not have joined the aggregation *i.e.*, $\pi_t^i(\mathbf{d}_t) \leq \theta(d_t^i)$.

Axiom 4 (Budget Balance): The cost allocation rule would be such that the sum of allocated costs must be equal to the net deviation cost *i.e.*, $\sum_{i\in N} \pi_t^i(\mathbf{d}_t) = \theta(d_t^{\mathcal{N}})$.

Axiom 5 (Standalone Cost Principle): For every aggregation $\mathcal{S} \subset \mathcal{N}$,

$$\sum_{i\in S} \pi_t^i(\mathbf{d}_t) \leq \theta(d_t^{\mathcal{S}}), \tag{3}$$

where $d_t^{\mathcal{S}} = \sum_{i\in\mathcal{S}} d_t^i$ is the net deviation of the subset of agents $\mathcal{S}$.

A cost causation based allocation as proposed by [15] should follow the general axioms of equity, monotonicity, individual rationality and budget balance, but not necessarily the standalone cost principle. However, not every allocation rule satisfying these four axioms follows the cost causation principle, because they do not explicitly take into account whether agents are causing or mitigating costs. Let us begin by formally defining who is causing and mitigating cost in our framework.

Definition 11 (Cost causation and mitigation): Let $d_t^{\mathcal{N}} = \sum_{i\in\mathcal{N}} d_t^i$ be the net deviation of a group of $\mathcal{N}$ agents. At time t, it is said that agent i is causing cost if $d_t^{\mathcal{N}} \neq 0$ and $d_t^i/d_t^{\mathcal{N}} > 0$, and is mitigating cost if $d_t^{\mathcal{N}} \neq 0$ and $d_t^i/d_t^{\mathcal{N}} < 0$.

The previous definition establishes a natural way to define cost mitigation. If an agent's deviation has opposite sign with respect to the net deviation, then this agent contributes to reduce the deviation. Now, we are ready to introduce two new cost causation based axioms: *penalty for cost causation* and *reward for cost mitigation*.

Axiom 6 (Penalty for causing cost): Those individuals causing cost should pay for it, *i.e.*, let $\theta(d_t^{\mathcal{N}}) \neq 0$, then $\pi_t^i(\mathbf{d}_t)/\theta(d_t^{\mathcal{N}}) > 0$ for any $i \in \mathcal{N}$ such that $d_t^i/d_t^{\mathcal{N}} > 0$.

Axiom 7 (Reward for cost mitigation): Those individuals mitigating cost should be rewarded, *i.e.*, let $\theta(d_t^{\mathcal{N}}) \neq 0$, then $\pi_t^i(\mathbf{d}_t)/\theta(d_t^{\mathcal{N}}) < \pi_t^j(\mathbf{d}_t)/\theta(d_t^{\mathcal{N}})$ for any $i, j \in \mathcal{N}$ such that $d_t^i/d_t^{\mathcal{N}} < 0$ and $d_t^j/d_t^{\mathcal{N}} \geq 0$.

The cost causation based tariffs [15] are defined by declarative principles, and they do not have a one to one correspondence axioms, but the combination of the axioms provide all of the principles. Principles 2c, 2d, 4a and 4b promote the aggregation of agents to improve the system integrity and reduce the costs and can be considered by axiom 3. Principles 2a and 2b are given by axioms 6 and 7, respectively. Principle 5 is provided by axiom 4. Principle 6 is equivalent to axioms 1 and 2, and the combination of axioms 1 to 4 guarantee principle 3.

Using the previously introduced axioms, an allocation that holds the statements proposed in [15] is called a *cost causation based* allocation rule and is formally defined as follows:

Definition 12 (Cost causation based allocation rule): A cost allocation rule is said to be a *cost causation based allocation* rule if it satisfies Axioms 1–4 and 6–7.

Some well-known cost allocation rules are the *proportional rule*, the *Shapley value*, and the *nucleolus* [23]. The proportional rule satisfies equity, monotonicity, individual rationality and budget balance, but not necessarily the standalone cost principle. The Shapley value does not necessarily satisfy monotonicity and the standalone cost principle. If the cost sharing problem can be modeled as a balanced cooperative game, then any allocation in the core of the game satisfies the standalone cost principle. In fact, the core is the set of all allocations satisfying the axioms of *budget balance* and *standalone cost principle*. Thus, a stabilizing cost allocation can be equivalently defined in an axiomatic way as follows:

Definition 13 (Stabilizing cost allocation rule): A cost allocation rule is said to be stabilizing if it satisfies the axioms of *budget balance* and *standalone cost principle*.

III. The Cost Allocation Problem for an Aggregation of Renewable Energy Producers

A. Problem Formulation

Consider a set of N independent renewable energy producers (REPs) indexed by $i \in \mathcal{N} := \{1, 2, \ldots, N\}$. Each REP $i \in \mathcal{N}$ has a nameplate production capacity W_i. The power produced by a subset $\mathcal{S} \subseteq \mathcal{N}$ of REPs at time t is modeled as a scalar valued random variable $w_t^{\mathcal{S}}$ with support $[0, W^{\mathcal{S}}]$, where $W^{\mathcal{S}}$ is the nameplate capacity of the subset $\mathcal{S}$, *i.e.*, $W^{\mathcal{S}} = \sum_{i\in\mathcal{S}} W^i$. The REPs offer their contractual power generation in a competitive two-settlement market system consisting of an *ex-ante* forward market and an *ex-post* imbalance mechanism to penalize uninstructed contract deviations. The price in *ex-ante* forward market is denoted by p and is assumed to be constant and known for every REP.[1] The penalty prices are modeled by random variables (q, λ), where q is the shortfall penalty price and λ is the surplus penalty price. Each REP calculate its own power contract C^i. Each of them has its own formula of power contract which may depend on its probability distribution function of power generation, p, estimated values of λ, q, cost allocation rule and many other factors. We assume that each REP does not communicate

[1] It means that either all the REPs are in the same bus or the network is not congested.

any information with other REPs to compute its own contract. This can be interpreted as an *ex-post* coalition where the REPs only share the net imbalance cost. The problem of *ex-ante* cooperation has been analyzed in [17] where the optimal joint contract that maximize the expected profit is obtained and the optimal expected profit is allocated among the members of the coalition.

Let $\mathcal{S} \subseteq \mathcal{N}$ denote a coalition of REPs. The total power contract of the coalition $\mathcal{S}$ is $C^{\mathcal{S}} = \sum_{i \in \mathcal{S}} C^i$. Let $w_t^{\mathcal{S}}$ denote the actual power produced by the coalition $\mathcal{S}$ at time $t \in [t_0, t_1]$ that is given by the sum of the individual powers, *i.e.*, $w_t^{\mathcal{S}} = \sum_{i \in \mathcal{S}} w_t^i$. The realized profit of $\mathcal{S}$ within a time interval $[t_0, t_1]$ is

$$pC^{\mathcal{S}}(t_1 - t_0) - \int_{t_0}^{t_1} \theta(d_t^{\mathcal{S}}, q, \lambda) dt, \tag{4}$$

where

$$d_t^{\mathcal{S}} = w_t^{\mathcal{S}} - C^{\mathcal{S}}, \tag{5}$$

$$\theta(d_t^{\mathcal{S}}, q, \lambda) = \lambda(d_t^{\mathcal{S}})_+ + q(-d_t^{\mathcal{S}})_+, \tag{6}$$

and $x_+ = \max\{x, 0\}$. If the joint deviation cost is not shared among the REPs and each REP is penalized based on its individual deviation, the realized profit of a REP is

$$pC^i(t_1 - t_0) - \int_{t_0}^{t_1} \theta(d_t^i, q, \lambda) dt. \tag{7}$$

Our problem is to distribute the joint profit among the individual REPs. The revenue obtained by selling the contracts $\{C^i : i \in \mathcal{N}\}$ does not change by aggregation, because the joint contract is the sum of the individual contracts, consequently the profit allocation problem is equivalent to allocating the net deviation cost $\{\theta(d_t^{\mathcal{S}}) : t \in [t_0, t_1]\}$. Moreover, the cost function (6) satisfies Assumption 1 because the operator $(\cdot)_+$ is subadditive and the problem can be cast in the general framework defined in Section II.

B. The Cost Allocation Problem

Let us define the deviation vector as

$$\mathbf{d}_t = \mathbf{w}_t - \mathbf{C}, \tag{8}$$

where $\mathbf{w}_t$ and $\mathbf{C}$ are the vector of realized power productions at time $t \in [t_0, t_1]$ and the vector of contracts for each REP, respectively.

Definition 14 (Joint cost and allocation): The joint cost incurred by the aggregation of REP within the interval $[t_0, t_1]$ is given by

$$\int_{t_0}^{t_1} \theta_t(d_t^{\mathcal{N}}, q, \lambda) dt, \tag{9}$$

where $d_t^{\mathcal{N}} = \sum_{i=1}^{\mathcal{N}} d_t^i$, and the individual allocation of the joint cost is given by

$$\int_{t_0}^{t_1} \pi_t^i(\mathbf{d}_t, q, \lambda) dt, \tag{10}$$

where $\{\pi_t^i(\mathbf{d}_t, q, \lambda) : i \in \mathcal{N}\}$ defines the cost allocation rule at time $t \in [t_0, t_1]$.

We partition the set $\mathcal{N}$ of REPs into two sets depending on if they have power production surplus or shortfall at time $t \in [t_0, t_1]$.

$$\mathcal{P}_t = \{i \in \mathcal{N} \mid d_t^i \geq 0\}, \tag{11}$$

$$\mathcal{Q}_t = \{i \in \mathcal{N} \mid d_t^i < 0\}. \tag{12}$$

Also we define $d_t^{\mathcal{P}} = \sum_{i \in \mathcal{P}_t} d_t^i$, $d_t^{\mathcal{Q}} = \sum_{i \in \mathcal{Q}_t} d_t^i$.

C. Previously Proposed Cost Allocation Rules

The cost allocation problem for an aggregation of REPs have been recently studied. Here we present two allocation rules recently developed for this problem and prove that they do not satisfy the cost causation principle. Then, we develop new cost causation based allocations.

a) The Proportional Allocation: It was proposed in [18] complying with the axioms of equity, monotonicity, individual rationality and budget balance. This cost allocation is given by

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = \frac{\theta(d_t^i, q, \lambda)}{\sum_{k \in \mathcal{N}} \theta(d_t^k, q, \lambda)} \theta(d_t^{\mathcal{N}}, q, \lambda), \ i \in \mathcal{N}. \tag{13}$$

b) The Statistically Robust Cost Allocation: It was proposed in [19] to satisfy the axioms of equity, individual rationality and budget balance. This cost allocation is as follows:

1) No net deviation ($d_t^{\mathcal{N}} = 0$):

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = 0, \qquad i \in \mathcal{N}. \tag{14}$$

2) Net shortfall ($d_t^{\mathcal{N}} < 0$): Let a be the unique solution of the equation $\sum_{i \in \mathcal{Q}_t} \min(a, |d_t^i|) = d_t^{\mathcal{P}}$. Then

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = 0, \qquad i \in \mathcal{P}_t, \tag{15}$$

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = -q(|d_t^i| - \min(a, |d_t^i|)), \quad i \in \mathcal{Q}_t. \tag{16}$$

3) Net surplus ($d_t^{\mathcal{N}} \geq 0$): Let b be the unique solution of the equation $\sum_{i \in \mathcal{P}_t} \min(b, d_t^i) = |d_t^{\mathcal{Q}}|$. Then

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = \lambda(d_t^i - \min(b, d_t^i)), \quad i \in \mathcal{P}_t, \tag{17}$$

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = 0, \qquad i \in \mathcal{Q}_t. \tag{18}$$

It is interesting to study if these allocations satisfy the cost causation principle. We prove in the following theorems that none of them is a cost causation based allocation.

Theorem 1: The proportional cost allocation rule is not a cost causation based allocation.

Proof: Let $\{\pi_t^i(\mathbf{d}_t, q, \lambda)\}$ denote the proportional allocation defined by equation (13). Let the aggregated deviation $d_t^{\mathcal{N}} \neq 0$ and $i, j \in \mathcal{N}$ be such that $d_t^i / d_t^{\mathcal{N}} < 0$, $d_t^j / d_t^{\mathcal{N}} \geq 0$ and $|d_t^i| > |d_t^j|$, then agent i is mitigating deviation cost and should be rewarded. However if $q = \lambda$, $\pi^i(\mathbf{d}_t, q, \lambda) > \pi^j(\mathbf{d}_t, q, \lambda)$. Consequently the proportional allocation is not a cost causation based allocation. ■

Theorem 2: The statistically robust cost allocation is not a cost causation based allocation.

Proof: Let $\{\pi_t^i(\mathbf{d}_t, q, \lambda)\}$ denote the statistically robust cost allocation given by equations (14)–(18). Let $d_t^{\mathcal{N}} > 0$ and $i \in \mathcal{P}_t$ be such that $\min(b, d_t^i) = d_t^i > 0$, then $\pi^i(\mathbf{d}_t, q, \lambda) = 0$. By analogy, let $d_t^{\mathcal{N}} < 0$ and $i \in \mathcal{Q}_t$ be such that $\min(a, |d_t^i|) = |d_t^i| > 0$, then $\pi^i(\mathbf{d}_t, q, \lambda) = 0$. In both cases, the agent i is not penalized for causing cost, and this proves that the statistically robust allocation is not a cost causation based allocation. ■

We conclude that the cost allocation rules in the literature for sharing the cost of power production deviation by an aggregation of REPs are not cost causation based allocation rules.

D. Conditions for a Cost-Causation-Based Allocation

A cost causation based allocation as defined in Definition 12 satisfies the following axioms: equity, monotonicity, individual rationality, budget balance, penalty for cost causation and reward for cost mitigation. We distinguish three cases: zero net deviation, positive net deviation and negative net deviation.

1) Zero net deviation ($d_t^{\mathcal{N}} = 0$): It implies $\theta(d_t^{\mathcal{N}}) = 0$. By applying budget balance and individual rationality axioms for any possible nonnegative penalty prices q and λ,

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = 0, \qquad i \in \mathcal{N}. \tag{19}$$

2) Positive net deviation ($d_t^{\mathcal{N}} > 0$): The individuals in $\mathcal{P}_t$ will be penalized for cost causation. The individuals in $\mathcal{Q}_t$ will be rewarded for cost mitigation. By applying equity and monotonicity axioms:

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = k_t^1 d_t^i, \qquad i \in \mathcal{P}_t, \tag{20}$$

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = k_t^2 d_t^i, \qquad i \in \mathcal{Q}_t, \tag{21}$$

where k_t^1 and k_t^2 are per unit penalty cost and reward. From budget balance axiom,

$$k_t^1 \sum_{i \in \mathcal{P}_t} d_t^i + k_t^2 \sum_{i \in \mathcal{Q}_t} d_t^i = \lambda d_t^{\mathcal{N}}. \tag{22}$$

3) Negative net deviation ($d_t^{\mathcal{N}} < 0$): The individuals in $\mathcal{P}_t$ will be rewarded for cost mitigation. The individuals in $\mathcal{Q}_t$ will be penalized for cost causation. By applying equity and monotonicity axioms:

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = k_t^3 d_t^i, \qquad i \in \mathcal{P}_t, \tag{23}$$

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = k_t^4 d_t^i, \qquad i \in \mathcal{Q}_t, \tag{24}$$

where k_t^3 and k_t^4 are per unit penalty cost and reward. From budget balance axiom,

$$k_t^3 \sum_{i \in \mathcal{P}_t} d_t^i + k_t^4 \sum_{i \in \mathcal{Q}_t} d_t^i = -q d_t^{\mathcal{N}}. \tag{25}$$

Conditions (20)–(25) define a family of cost allocation rules depending on the values of the per unit penalty costs and rewards $(k_t^1, k_t^2, k_t^3, k_t^4)$ that can be obtained solving the linear equations (22) and (25) at time t. But there are only two ways the parameters k_t^1, k_t^2, k_t^3 and k_t^4 can be chosen such that Axioms 6 and 7 are satisfied. In the first case, we have $k_t^2 = k_t^3 = 0$. It means that the individuals contributing to reduce the net deviation are exempted to pay any costs for their deviations. The values of k_t^1 and k_t^4 in this case are directly obtained as $k_t^1 = \lambda d_t^{\mathcal{N}} / \sum_{i \in \mathcal{P}_t} d_t^i$ and $k_t^4 = -q d_t^{\mathcal{N}} / \sum_{i \in \mathcal{Q}_t} d_t^i$. In the second case, $k_t^1 = k_t^2 = \lambda$ and $k_t^3 = k_t^4 = -q$. Here the agents contributing to reduce the net deviation are rewarded at the same rate at which the agents that contribute to generate the net deviation are penalized. This allocation is very attractive and easy to compute because the reward and penalty prices are independent of the deviations and take the same values as the market deviation penalty prices. These two allocation rules are formally defined as follows.

Definition 15 (Cost causation based allocation rule with zero reward): The cost-causation based allocation with zero reward is defined as

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = \lambda \frac{(d_t^i)_+}{\sum_{i \in \mathcal{N}} (d_t^i)_+} (d_t^{\mathcal{N}})_+ + q \frac{(-d_t^i)_+}{\sum_{i \in \mathcal{N}} (-d_t^i)_+} (-d_t^{\mathcal{N}})_+, \; i \in \mathcal{N}. \tag{26}$$

The above allocation rule can also be expressed in a simplified way as follows.

1) If $d_t^{\mathcal{N}} = 0$:

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = 0, \qquad i \in \mathcal{N}. \tag{27}$$

2) If $d_t^{\mathcal{N}} < 0$:

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = 0, \qquad i \in \mathcal{P}_t, \tag{28}$$

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = -q \frac{d_t^{\mathcal{N}}}{\sum_{i \in \mathcal{Q}_t} d_t^i} d_t^i, \qquad i \in \mathcal{Q}_t. \tag{29}$$

3) If $d_t^{\mathcal{N}} > 0$:

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = \lambda \frac{d_t^{\mathcal{N}}}{\sum_{i \in \mathcal{P}_t} d_t^i} d_t^i, \qquad i \in \mathcal{P}_t, \tag{30}$$

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = 0, \qquad i \in \mathcal{Q}_t. \tag{31}$$

Definition 16 (Cost causation based allocation rule with nonzero reward): The cost causation based allocation with nonzero reward is defined as

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = \lambda \frac{d_t^i}{|d_t^{\mathcal{N}}|} \left(d_t^{\mathcal{N}}\right)_+ - q \frac{d_t^i}{|d_t^{\mathcal{N}}|} \left(-d_t^{\mathcal{N}}\right)_+, \; i \in \mathcal{N}. \tag{32}$$

We can also write this allocation rule in a simplified way as follows:

$$\pi_t^i(\mathbf{d}_t, q, \lambda) = \begin{cases} 0, & \text{if } d_t^{\mathcal{N}} = 0 \\ -q d_t^i, & \text{if } d_t^{\mathcal{N}} < 0 \\ \lambda d_t^i, & \text{if } d_t^{\mathcal{N}} > 0 \end{cases}$$

It is interesting to check if the two developed allocations satisfy also the standalone cost principle. In the following theorems, we prove that only the cost causation based allocation with nonzero reward satisfies the standalone cost principle.

Theorem 3: The cost causation based allocation rule with zero reward does not satisfy the standalone cost principle.

Proof: The standalone cost principle is satisfied for the cost allocation $\pi_t(\mathbf{d}_t)$ if for any coalition $\mathcal{S} \subset \mathcal{N}$, the excess $e(\pi_t(\mathbf{d}_t), \mathcal{S}) = \sum_{i \in \mathcal{S}} \pi_t^i(\mathbf{d}_t) - \theta(d_t^{\mathcal{S}})$ is nonnegative. Let us assume that the deviations $\mathbf{d}_t \in \mathbb{R}^N$ for a group of N agents are such that $d_t^i = -d_t^j$, $d_t^{\mathcal{N}} \neq 0$ and $d_t^i / d_t^{\mathcal{N}} > 0$, then the cost function $\theta(d_t^{\mathcal{N}}) > 0$ and the cost causation rule with zero rewards provides $\pi^i(\mathbf{d}_t) > 0$ and $\pi^j(\mathbf{d}_t) = 0$. The excess of the coalition $\{i, j\}$ is negative because $e(\pi, \{i, j\}) =$

$\theta(d_t^i + d_t^j) - \pi_t^i(\mathbf{d}_t) - \pi_t^j(\mathbf{d}_t) = -\pi_t^j(\mathbf{d}_t^i) < 0$ and the allocation does not satisfy the standalone cost principle. ■

Theorem 4: The cost causation based allocation rule with nonzero reward satisfies the standalone cost principle and consequently is an allocation in the core of the cooperative game.

Proof: For the cost causation based allocation with nonnegative reward, the excess is

$$\begin{aligned} e\left(\pi_t(\mathbf{d}_t), \mathcal{S}\right) &= \theta\left(d_t^{\mathcal{S}}\right) - \lambda \frac{d_t^{\mathcal{S}}}{|d_t^{\mathcal{N}}|}\left(d_t^{\mathcal{N}}\right)_+ + q\frac{d_t^{\mathcal{S}}}{|d_t^{\mathcal{N}}|}\left(-d_t^{\mathcal{N}}\right)_+ \\ &= \lambda\left(\left(d_t^{\mathcal{S}}\right)_+ - \frac{d_t^{\mathcal{S}}}{|d_t^{\mathcal{N}}|}\left(d_t^{\mathcal{N}}\right)_+\right) \\ &\quad + q\left((-d_t^{\mathcal{S}})_+ + \frac{d_t^{\mathcal{S}}}{|d_t^{\mathcal{N}}|}(-d_t^{\mathcal{N}})_+\right). \end{aligned}$$

We can distinguish two cases: if $d_t^{\mathcal{S}}/d_t^{\mathcal{N}} \geq 0$ then $e(\pi_t(\mathbf{d}_t), \mathcal{S}) = 0$, otherwise $e(\pi_t(\mathbf{d}_t), \mathcal{S}) = (\lambda + q)|d_t^{\mathcal{S}}|$. Thus, the excess is always nonnegative and the allocation satisfies the axiom of standalone cost principle. Since the allocation also satisfies the budget balance axiom, it lies in the core of the cooperative game. ■

The cost causation based allocation with nonzero reward is a very appealing mechanism because in addition to complying with the cost causation principle, it also lies in the core of the corresponding cooperative game. It means that no player in the game is dissatisfied with this allocation because no player can reduce its allocated cost by breaking up the coalition and forming a subcoalition. Moreover, this allocation is easy to compute and computation complexity does not increase with the number of individuals.

IV. The General Cost Allocations and the Cost Causation Principle

A. General Cost-Sharing Rules

The general cost sharing problem deals with a scenario of a finite number of agents that have to allocate costs coming from multiple heterogeneous causes using only the information of the cost function. There is no completely satisfactory solution to this problem, but several rules based on the general axioms for the one cost cause case have been proposed [23], [29]. Three of the most popular are: The *Shapley-Shubik* (SS), the *Aumann-Shapley* (AS) and the *Friedman-Moulin* (FM) rules. In this section, we derive expressions of these general allocation rules for our problem set-up of sharing a deviation cost and analyze if these allocations comply with the cost causation principle.

In the general allocation problem, there is a nonnegative vector of cost causes, $\mathbf{m} \in \mathbb{R}_+^N$ and a function $\theta_c : \mathbb{R}_+^N \to \mathbb{R}$ that assigns a cost to each vector of causes. The cost function satisfies $\theta_c(\mathbf{0}) = \mathbf{0}$ and is assumed to be continuous and piecewise differentiable.

The allocation problem is to split the cost θ_c among the different causes. Let $\nabla\theta_c(\mathbf{m})$ denote the gradient of the cost function with respect to the cause variables evaluated at the coordinate point $\mathbf{m}$ and $\nabla^i\theta_c(\mathbf{m})$ its ith component. A first straightforward approach to allocate the cost is to define a cooperative game where the players are the different cost causes and the cost function is θ_c, then the allocation given by the Shapley value for this cooperative game provides a mechanism that is called *Shapley-Shubik allocation rule*.

Definition 17: The Shapley-Shubik allocation rule is defined as follows:

$$\pi_{\mathrm{SS}}^i(\mathbf{m}, \theta_c) = \sum_{\mathcal{S}\subset\mathcal{N}\backslash i} \frac{|\mathcal{S}|!(N - |\mathcal{S}| - 1)!}{N!} v^i(\mathcal{S}),\ i \in \mathcal{N}, \quad (33)$$

where

$$\begin{aligned} v^i(\mathcal{S}) &= \theta_c(\mathbf{m}^{\mathcal{S}\cup\{i\}}) - \theta_c(\mathbf{m}^{\mathcal{S}}), \qquad i \in \mathcal{N}\backslash\mathcal{S}, \\ \mathbf{m}^{\mathcal{S}} &= \sum_{i\in\mathcal{S}} m^i \mathbf{e}_i, \end{aligned}$$

and $\mathbf{e}_i$ is the ith column of the identity matrix of size N.

The Shapley-Shubik allocation rule has two major drawbacks. First, it does not explicitly use the properties of the cost function. Second, it is not computationally amenable for a game with a large number of players N because it requires to calculate the costs associated to 2^N coalitions.

An alternative approach is to decompose the cost function using its differentiability properties. A class of allocation rules can be defined by considering a differentiable cost function on the nonnegative orthant $\mathbb{R}_+^N$. A vector field can be obtained from this cost function by using the gradient operator. The value of the cost function at a given coordinate point $\mathbf{m} \in \mathbb{R}_+^N$ in the geometric space of cost causes can be obtained by integrating the gradient vector field along a continuous path connecting the origin with the point $\mathbf{m}$.

Let us consider an arbitrary piecewise smooth path $\Gamma := \Gamma_1 \circ \Gamma_2 \circ \ldots \circ \Gamma_\ell$, where $\Gamma_k := \{\mathbf{z}_k(\tau) : \tau \in [\tau_{k-1}, \tau_k]\}$, and $\tau_0 = 0$, $\tau_\ell = 1$ such that $\mathbf{z}_1(0) = 0$ and $\mathbf{z}_\ell(1) = \mathbf{m}$. The value of the cost function at the coordinate point $\mathbf{m}$ is given by

$$\theta_c(\mathbf{m}) = \sum_{i\in\mathcal{N}}\sum_{k=1}^{\ell}\int_{\tau_{k-1}}^{\tau_k} \nabla^i\theta_c(\mathbf{z}(\tau))\nabla z_k^i(\tau)d\tau. \quad (34)$$

The previous expression provides a natural decomposition of the cost function into the coordinates of its cost causes. Then, a generic allocation rule is defined as follows:

$$\pi^i(\mathbf{m}, \theta_c) = \sum_{k=1}^{\ell}\int_{\tau_{k-1}}^{\tau_k} \nabla^i\theta_c(\mathbf{z}(\tau))\nabla z_k^i(\tau)d\tau,\ i \in \mathcal{N}. \quad (35)$$

Different allocation rules are obtained by choosing different integration paths Γ. The Aumann-Shapley and the Friedman-Moulin rules belong to this class of allocation rules. The Aumann-Shapley rule is obtained when Γ is the line connecting $\mathbf{0}$ and $\mathbf{m}$, while the Friedman-Moulin rule is obtained when Γ is a piecewise linear trajectory that joins $\mathbf{0}$ and $\mathbf{m}$ by raising all coordinates at the same rate and freezing each coordinate once it reaches its final value.

Definition 18: The Aumann-Shapley allocation rule is defined as follows:

$$\pi_{\mathrm{AS}}^i(\mathbf{m}, \theta_c) = m^i \int_0^1 \nabla^i\theta_c(\mathbf{z}(\tau))d\tau,\ i \in \mathcal{N}, \quad (36)$$

where $\left\{z^\ell(\tau) = m^\ell \tau : \ell \in \mathcal{N}\right\}$.

Definition 19: The *Friedman-Moulin* allocation rule is defined as follows:

$$\pi^i_{\mathrm{FM}}(\mathbf{m}, \theta_c) = m^i \int_0^1 \nabla^i \theta_c(\mathbf{z}(\tau)) d\tau,\ i \in \mathcal{N}, \tag{37}$$

where $\left\{z^\ell(\tau) = \min\{m^i\tau, m^\ell\},\ \ell \in \mathcal{N}\right\}$.

We prove in the following lemma that the both the Aumann-Shapley and Friedman-Moulin rules result in the same allocation if the cost function is linear in the cost causes.

Lemma 1: The Friedman-Moulin allocation rule is equivalent to the Aumann-Shapley allocation rule whenever the cost function is linear in the cost causes.

Proof: A cost function that is linear in the cost causes has a constant gradient. Consequently, both the Aumann-Shapley and Friedman-Moulin allocation rules are equal to

$$\pi^i_{\mathrm{AS}} = \pi^i_{\mathrm{FM}} = m^i \nabla^i \theta_c(\mathbf{z}(\tau)), i \in \mathcal{N}. \tag{38}$$

■

Our deviation cost allocation problem can be cast in this general framework because we have N agents that generate individual costs associated to their particular deviations.

B. The General Allocation Rules for the Net Deviation Cost

In this section, we will analyze if the general cost allocation rules satisfy the cost causation based axioms.

For our problem, the cost is obtained when the deviations are realized. We now consider an equivalent cost function that takes into account the deviation signs and is linear in the causes of cost. We refer to it as the *a posteriori* cost function. This equivalent cost function will be used to allocate the cost to the different causes.

Let $\mathbf{d}_t$ be the vector of realized deviations. By fixing the sign of the realized deviations and the sign of the net deviation $d_t^{\mathcal{N}}$, we define the following *a posteriori* cost function

$$\theta_c(\mathbf{m}) = \sum_{i \in \mathcal{N}} s^i m^i \tag{39}$$

where the value of s^i depends on the sign of the realized deviations. If the net deviation is nonnegative, then $s^i = \lambda$ if $d_t^i \geq 0$ and $s_t^i = -\lambda$ if $d_t^i < 0$. However, if the net deviation is negative, then $s^i = q$ if $d_t^i \geq 0$ and $s_t^i = -q_i$ if $d_t^i < 0$. Thus, we define the row vector $\mathbf{s}$, such that

$$s^i = \frac{d_t^i}{|d_t^i|}\left(\lambda \frac{(d_t^{\mathcal{N}})_+}{|d_t^{\mathcal{N}}|} - q\frac{(-d_t^{\mathcal{N}})_+}{|d_t^{\mathcal{N}}|}\right),\ i \in \mathcal{N}. \tag{40}$$

The cost associated to the deviation vector $\mathbf{d}_t$ is given by $\theta(\mathbf{d}_t) = \theta_c(\mathsf{abs}(\mathbf{d}_t))$ where $\mathsf{abs}(\mathbf{x})$ for a vector $\mathbf{x}$ is another vector of the same dimension whose ith entry is the absolute value of the ith entry of $\mathbf{x}$. The *a posteriori* cost function θ_c is a linear function on the vector of cost causes $\mathbf{m} \in \mathbb{R}_+^N$. Therefore, it is a continuously differentiable function on the positive orthant $\mathbb{R}_+^N$ and its gradient is given by:

$$\nabla\theta_c(\mathbf{m}) = \mathbf{s},\ \mathbf{m} \in \mathbb{R}_+^N \tag{41}$$

Using the cost function θ_c and its gradient, we can obtain the general allocation rules and study if they satisfy the cost causation axioms. We begin by showing that the Shapley-Shubik general rule is not a cost causation based allocation, because it inherits the properties of the Shapley value and therefore, does not hold monotonicity. Consider a three dimensional deviation vector $\mathbf{d}_t$. Assuming that the penalty prices are both equal to one, *i.e.*, $q = \lambda = 1$ and the deviations satisfy

$$d_t^1 \geq 0, \qquad d_t^2 \geq 0, \qquad d_t^3 < 0,$$
$$d_t^1 + d_t^2 \geq 0, \qquad d_t^2 + d_t^3 < 0, \qquad d_t^1 + d_t^3 < 0,$$
$$d_t^1 + d_t^2 + d_t^3 \geq 0.$$

The Shapley-Shubik allocation is given by

$$\pi_t^1(\mathbf{d}_t) = \frac{2}{3}\alpha, \qquad \pi_t^2(\mathbf{d}_t) = \frac{2}{3}\alpha, \qquad \pi_t^3(\mathbf{d}_t) = -\frac{1}{3}\alpha,$$

where $\alpha = d_t^1 + d_t^2 + d_t^3$. The allocation does not satisfy the monotonicity axiom when $d_t^1 \neq d_t^2$ because $\pi_t^1 = \pi_t^2$ for any $d_t^1 \geq 0$ and $d_t^2 \geq 0$. Consequently, we have the following result.

Observation: The Shapley-Shubik rule is not a cost causation based allocation.

We analyze now the Aumann-Shapley and Friedman-Moulin rules under the cost causation principle. Since the cost function is linear in the cost causes, we prove that both rules provide the same allocation. Moreover, this allocation is equivalent to the allocation rule with nonzero reward that we derived in the previous section using exclusively the axiomatic formulation of the cost causation principle.

Theorem 5: The Aumann-Shapley rule and the Friedman-Moulin rule for the *a posteriori* cost function produce the same cost allocation and it is equivalent to the cost causation based allocation with nonzero reward.

Proof: The equivalency of the cost allocation obtained applying the Aumann-Shapley and the Friedman-Moulin rules follows by linearity of the cost function with respect to the cost causes. This allocation is easily obtained from the equation (38), where the gradient of the cost function is given by equation (41). The resulting allocation rule for $\mathbf{m} = \mathsf{abs}(\mathbf{d}_t)$ is given by

$$\begin{aligned}\pi_t^i(\mathbf{m}, \theta_c) &= |d_t^i| s^i \\ &= d_t^i \left(\lambda \frac{(d_t^{\mathcal{N}})_+}{|d_t^{\mathcal{N}}|} - q\frac{(-d_t^{\mathcal{N}})_+}{|d_t^{\mathcal{N}}|}\right)\end{aligned}$$

but this is precisely the cost causation based allocation with nonzero reward. ■

The previous theorem proves that the Aumann-Shapley rule for allocation of deviation costs, where the cost function is a piecewise linear function of the net deviation (6) is equivalent to the cost causation-based allocation with nonzero reward. Thus, it provides a computationally amenable allocation that follows the cost causation principle and lies in the core of the corresponding cooperative game.

V. Case Studies

In this section, we present two examples to illustrate the different cost allocation rules analyzed in this paper. The first

TABLE I
CONTRACTS, REALIZED WIND POWER, AND DEVIATIONS (MW)

i	C^i	w^i	d_i
1	200	210	10
2	120	140	20
3	260	250	-10
4	310	330	20
5	280	230	-50

example is a simple study for $N = 5$ wind farms and one hour. It has been design to easily show the properties of the different cost allocation rules. The second example uses real data of wind and prices of the Iberian electrical market (Spain and Portugal) and it demonstrates that the applicability of the cost causation based allocation rules.

A. A Simple Example

Consider $N = 5$ wind farms that inject power in the same bus of the grid. They decide to cooperate to reduce the imbalance cost by aggregation. The wind farms belong to different firms and they do not want to disclose their operational procedures and the statistical information and tools they use to forecast their own wind power production and the imbalance prices. For this reason, each wind farm decides to calculate its own optimal contract and they bid the sum of the individual contracts. Each wind farm uses its private estimated probability distribution of its production and the expected values of the deviation penalty prices to compute the contract $C^i(p)$ that optimizes its own expected profit for each price p using the formula given by [30]. They bid the aggregate contract $C^{\mathcal{N}}(p) = \sum_{i=1}^{5} C^i(p)$ in the day ahead market. When market closes, the clearing price is obtained. Let us assume that the clearing price is $p = \$50/\text{MW}$ for a given hour of the day, and the corresponding contracts for that hour are given in Table I. The wind power is realized at the delivery time for each wind farm and the deviations during that hour are calculated. The corresponding data are given in Table I.

The deviations are penalized at the penalty prices q and λ that are obtained after the deviations are known because the system operator has to compute the cost of compensating the imbalances. Let us assume that the penalty prices are $q = \lambda = \$100/\text{MW}$.

The results of the different allocation mechanisms are shown in Table II where the cost figures are shown in thousands of dollars accordingly to the following notation:

(a) Cost causation based allocation with zero reward.
(b) Cost causation based allocation with nonzero reward.
(c) Proportional allocation.
(d) Statistically robust allocation.
(e) Aumann-Shapley (and Friedman-Moulin) allocation.
(f) Shapley-Shubik allocation.

The key observations from this example are summarized as follows:

1) All the allocations satisfy equity because the REPs 2 and 4 have the same deviation and the allocated cost is the same.

TABLE II
DEVIATIONS (MW) AND ALLOCATED COSTS ($\times 10^3$ \$)

i	d^i	$\theta(d^i)$	**(a)**	**(b)**	**(c)**	**(d)**	**(e)**	**(f)**
1	10	1	0	-1	$\frac{1}{11}$	0	-1	$-\frac{1}{10}$
2	20	2	0	-2	$\frac{2}{11}$	0	-2	$-\frac{1}{10}$
3	-10	1	$\frac{1}{6}$	1	$\frac{1}{11}$	0	1	$\frac{2}{5}$
4	20	2	0	-2	$\frac{2}{11}$	0	-2	$-\frac{1}{10}$
5	-50	5	$\frac{5}{6}$	5	$\frac{5}{11}$	1	5	$\frac{9}{10}$

TABLE III
RELATIONSHIP BETWEEN ALLOCATIONS AND AXIOMS

Axioms	(a)	(b)	(c)	(d)	(e)	(f)
1	Y	Y	Y	Y	Y	Y
2	Y	Y	Y	N	Y	N
3	Y	Y	Y	Y	Y	Y
4	Y	Y	Y	Y	Y	Y
5	Y	Y	N	N	Y	N
6	Y	Y	N	N	Y	N
7	Y	Y	N	N	Y	N

2) The Shapley-Shubik allocation (f) does not satisfy the the axiom of monotonicity, because REPs 1 and 2 have different deviations, but they receive the same cost allocation.
3) All the allocations satisfy budget balance because the sum of the allocated costs equals the net deviation cost.
4) All the allocations satisfy individual rationality because the allocated cost to each REP never is greater that its individual deviation cost.
5) The proportional allocation (c) is not a cost causation based allocation because it penalizes the REPs that contribute to reduce the net deviation.
6) The statistically robust allocation (d) is not cost causation based because it does not penalize REP 3 that causes cost.
7) The cost causation based allocation with nonzero reward (b) is the same as the Aumann-Shapley (and Friedman-Moulin) allocation (e). Also the allocation can be verified to be lying in the core.

All these observations confirm the results obtained in the theoretical analysis developed in the previous sections. We also show in Table III the fulfilment of different axioms by the cost allocations analyzed in this example. In this table, ‘Y’ means the allocation satisfies the corresponding axiom, whereas ‘N’ means satisfaction is not assured.

Finally, in Table IV we show the realized profit for each wind farm with different allocation rules. The realized profit is obtained as the difference between the revenues by selling the contract C^i in the day ahead market and the allocated penalty cost for the joint deviation. The results are compared to the profit (ip) of each wind farm bidding individually in the day ahead market.

TABLE IV
REALIZED PROFIT FOR EACH WIND FARM ($\times 10^3$ \$)

i	(ip)	(a)	(b)	(c)	(d)	(e)	(f)
1	9	10	11	$\frac{109}{11}$	10	11	$\frac{101}{10}$
2	4	6	8	$\frac{64}{11}$	6	8	$\frac{61}{10}$
3	12	$\frac{77}{6}$	12	$\frac{142}{11}$	13	12	$\frac{63}{5}$
4	$\frac{27}{2}$	$\frac{31}{2}$	$\frac{35}{2}$	$\frac{337}{22}$	$\frac{31}{2}$	$\frac{35}{2}$	$\frac{78}{5}$
5	9	$\frac{79}{6}$	9	$\frac{149}{11}$	13	9	$\frac{131}{10}$

TABLE V
LOCATION AND CAPACITY OF THE WIND FARMS

Wind Farm	Latitude	Longitude	Capacity (MW)
1	36° 50′ 47″ N	2° 21′ 25″ W	130
2	42° 21′ 22″ N	3° 37′ 17″ W	200
3	43° 18′ 25″ N	8° 22′ 19″ W	150
4	36° 38′ 43″ N	6° 20′ 58″ W	100
5	41° 39′ 38″ N	1° 0′ 15″ W	150

B. A Study With Real Data

In this study, we have considered $N = 5$ wind farms located at different locations in Spain, see Table V. We use real data on wind velocity, prices and deviation penalties from the Iberian Peninsula (Spain and Portugal) electrical system. The prices have been obtained from the web page of the Spanish ISO [31] and corresponds to the DA market price p and the deviation penalties (q, λ) for each hour for two winter months in 2015-16. In the Iberian electrical market, contrary to US electrical markets, LMPs are not considered and the DA market price is the same for every bus of the electrical grid. Another feature of the Iberian market is that the ISO buys all the electrical production. If there is a surplus of power in the system, the ISO generates a nonnegative deviation price that is lower than the DA price market, and the surplus is paid at this price. The reason for this policy is that Spain has a large number of hydroelectric plants with pumped storage capability. Thus, the surplus production can be used for storage. In connection to our model, this means that the surplus penalty price λ is always nonpositive, and the cost of the positive deviation is the loss of income because this energy was paid at positive deviation price that is usually lower than the DA market price.

The wind speed measurements (from 2015/12/01 to 2016/01/29) have been obtained from the corresponding meteorological stations [32]. These wind speed surface measurements have been extrapolated to 90m of height and converted to energy using the power curve of a Vestas V52-850KW (IEC class I/II) wind turbine [33], as shown in Fig. 1.

Using the power production values obtained from the wind speeds records, we have obtained the estimated cumulative distribution function for each wind farm at each hour of the day. The empirical CDFs for hour 4 and 16 of a winter day are depicted in Fig. 2.

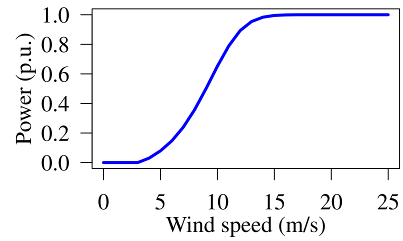

Fig. 1. Normalized power curve of a Vestas V52 850-kW wind turbine.

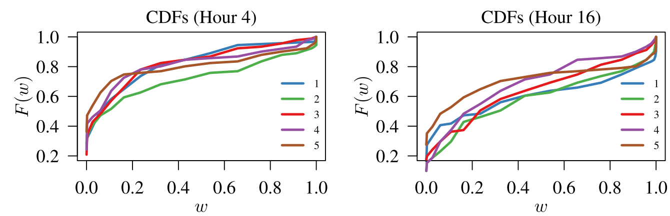

Fig. 2. Estimated CDFs for hours 4 and 16.

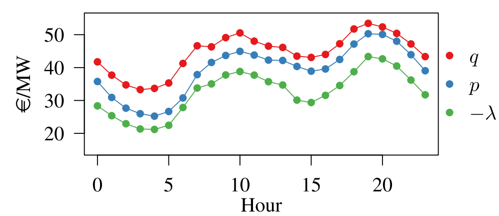

Fig. 3. Expected prices.

Each wind farm i can compute the contract that maximizes its expected profit by using the empirical CDF at a given hour and the information that it has about the DA market price and the deviation penalty prices as follows

$$C^i = F_i^{-1}(\gamma) \tag{42}$$

where F_i^{-1} is the quantile function associated to the empirical CDF F_i and γ is a ratio of the DA market price p and the conditioned expected deviation prices $\mu_q(p) = \mathbb{E}[q|p]$ and $\mu_\lambda(p) = \mathbb{E}[\lambda|p]$. The expression of γ is

$$\gamma = \frac{p + \mu_\lambda(p)}{\mu_q + \mu_\lambda(p)}. \tag{43}$$

This optimal contract formula that maximizes the expected profit was obtained in [30].

In the Iberian electrical market, the DA price and the penalty prices are strongly correlated as we can see in Fig. 3 where the mean values of p, q and $-\lambda$ are shown for each hour of 2015/2016 winter. Using this fact, we used linear regression to obtain expressions for the conditioned expected deviation prices $\mu_q(p)$ and $\mu_\lambda(p)$ for each hour of the day.

We assume that the wind energy producers know the DA price and they compute the individual contracts that maximize their expected benefit and the aggregated contract $C = \sum_{i \in \mathcal{N}} C^i$ is offered in the DA market. The individual optimal contracts and the aggregated contract are shown in Fig. 4 for hours 4 and 16. It is interesting to note that at hour 4 the expected shortfall

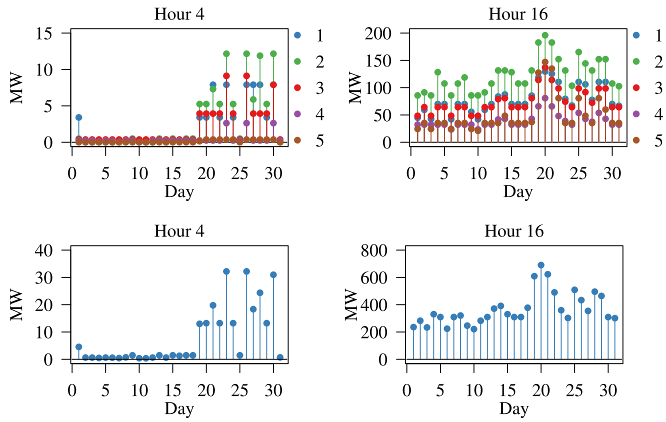

Fig. 4. Individual and aggregated contracts for hours 4 and 16.

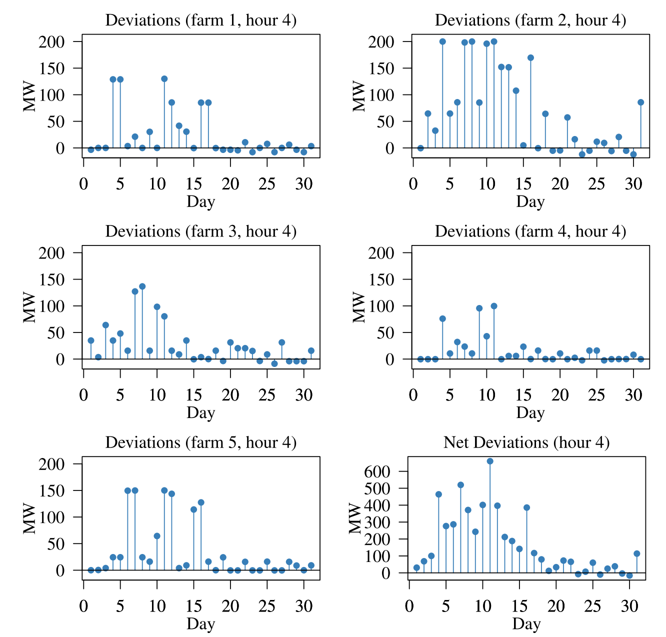

Fig. 5. Deviations for hour 4.

deviation price is large compared to the DA market price, but the surplus deviation price is not very different from the DA market price. Consequently, the wind producers decide to offer small contracts to cover from shortfall penalties. On the contrary, on hour 16, the ratio between the expected shortfall penalty price and the DA market price is not so high, while the surplus deviation prices is relatively lower. For this reason, the most important loss contribution occurs if they have to sell the surplus production as a deviation at a small price, and the expected profit is maximized for large contracts.

When actual wind is realized, the wind energy productions are delivered and the deviations are computed. These deviations are shown in Figs. 5 and 6 for each individual wind farm and for the aggregated contract at hours 4 and 16 of each day of January of 2016, using real data of wind at each wind farm location. Since the five wind farms offered their aggregated contract in the DA market as a coalition, they are penalized for the aggregated deviation at the deviation prices of each day. The

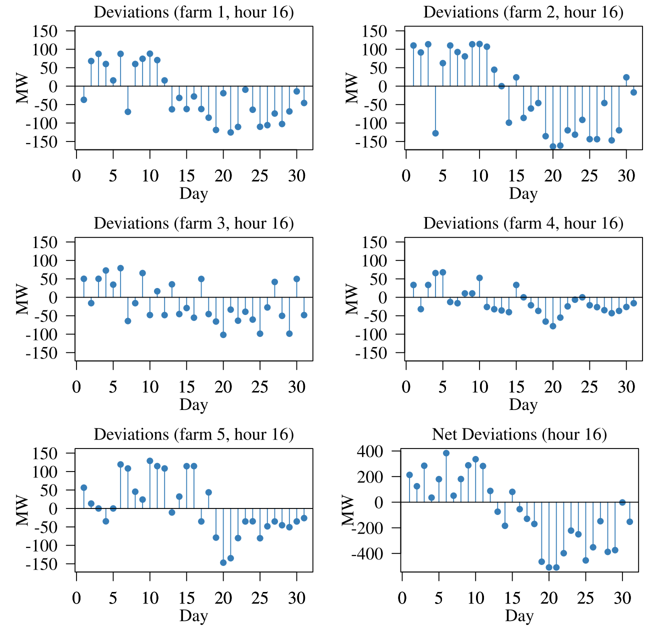

Fig. 6. Deviations for hour 16.

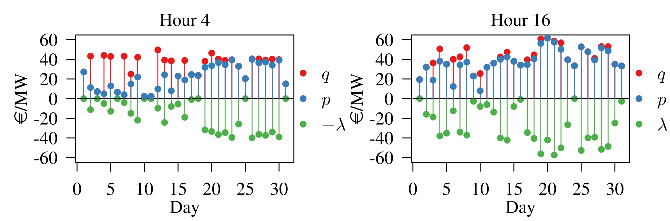

Fig. 7. DA market price p and deviation penalties q, λ.

actual DA market price and the deviation prices for hours 4 and 16 of every day of January of 2016 are shown in Fig. 7. Using the realized deviations and the actual penalty prices, we have computed the aggregated cost and the allocation of cost to each wind producer. We have compared three sharing rules: the cost causation based allocation with nonzero reward (CCBA-NR), the cost causation based allocation with zero reward (CCBA-ZR), and the Shapley-Shubik allocation (SSA). The deviation cost is allocated at each hour of the day, and the aggregated results for each day of January 2016 are shown in Fig. 8. Finally, a summary of the global results are shown in Table VII where the column with title NC represents the costs if there was no cooperation. Here, we show the monthly aggregated cost allocated to each wind producer. The aggregated cost corresponds to 744 allocations, each one for each hour of the month. The two last rows of this table present the aggregated worst-case excess (e^*) and the number of times that the worst case excess is negative. The cost causation based allocation with nonzero reward (CCBA-NR) is satisfactory for every coalition member for every hour. It is interesting to note that the Shapley-Shubik allocation (SSA) is not satisfactory for 385 hours of the month. Besides, the aggregated worst-case excess is much lower for the SSA than for the cost causation based allocation with zero reward (CCBA-ZR). In fact, the SSA is similar to the CCBA-NR as we can see in Fig. 8 and Table VII. In order to analyze this

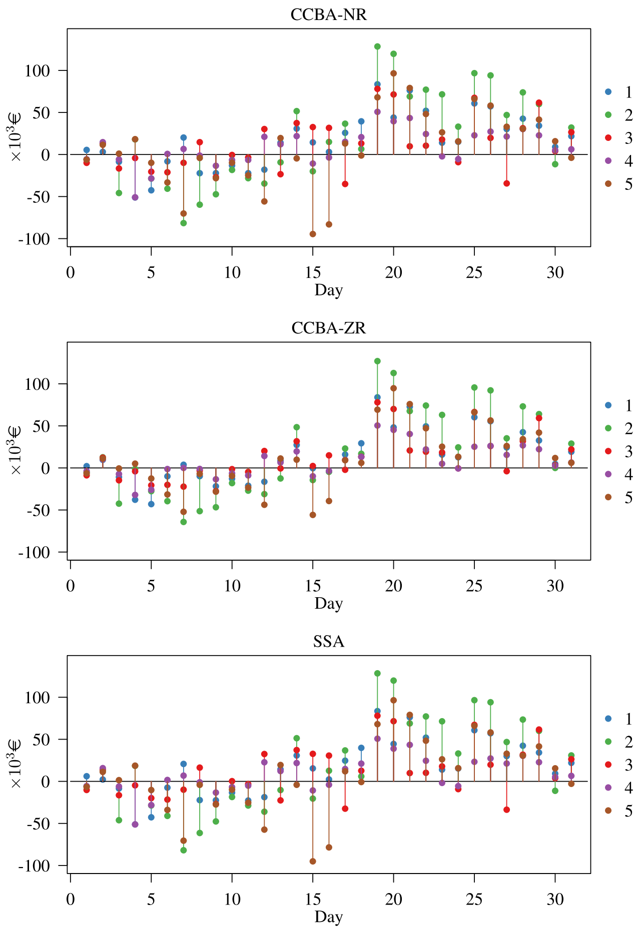

Fig. 8. Comparison of cost allocations.

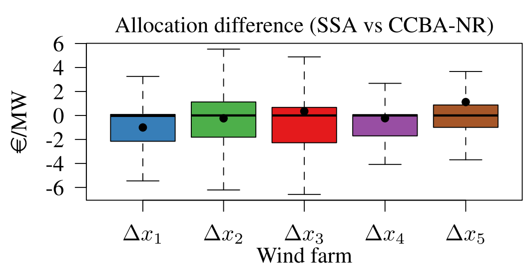

Fig. 9. Statistical analysis of the difference in SSA and CCBA-NR.

difference, we have performed a statistical study of the difference between both allocations for the 385 hours where the SSA is not satisfactory. A boxplot of the allocation difference normalized by the capacity of each wind farm is shown in Fig. 9 and the numerical values are given in Table VI. From this study we can see that the difference beetween both allocations is small, most of the times in the range of ±€6/MW. The dispersion is greater for wind farms 2 and 3 while the average error is larger for wind farms 1 and 4. However, there is no clear trend and the hourly dissatisfaction provided by the SSA rule is similarly distributed among the different wind farms.

This provides a solid reason to advocate for the CCBA-NR rule, because it always provides satisfactory allocations that are always in the core of the cooperative game and they are not far from the allocations provided by a highly recommended sharing

TABLE VI
STATISTICAL ANALYSIS OF THE DIFFERENCE IN SSA AND CCBA-NR (€/MW)

	Min	Q1	Q2	Mean	Q3	Max
Δx_1	−22.6667	−2.1525	−0.0326	−1.0008	0.0871	29.6134
Δx_2	−37.3530	−1.8104	0.0021	−0.2387	1.1274	29.1201
Δx_3	−28.9456	−2.2742	0.0000	0.3509	0.6742	50.9891
Δx_4	−11.6435	−1.7026	−0.0005	−0.2307	0.0665	29.0443
Δx_5	−23.9393	−0.9913	0.0000	1.1194	0.8739	49.7768

TABLE VII
SUMMARY OF COST ALLOCATIONS (IN €) FOR JANUARY 2016

	NC	CCBA-NR	CCBA-ZR	SSA
1	521567.40	489021.34	430514.80	488119.70
2	630155.83	612294.22	573373.32	600178.46
3	415919.02	356762.45	375600.51	363438.64
4	306467.51	266359.08	247373.95	271327.98
5	256348.84	213185.49	310760.00	214557.81
$\sum e^*$	–	0.00	−490801.25	−23079.26
$\#(e^* < 0)$	–	0	422	385

rule (the Shapley value) but at a much lower computational burden, because the complexity of the Shapley value increases exponentially with the cardinality of the coalition.

VI. CONCLUSION

We developed an axiomatic framework for cost causation based allocation of deviation costs. This framework can be applied to allocating the aggregated cost of a group of agents that cooperate in reducing the net deviation. This is a general class of problems with great application in power systems. We applied the framework to share the imbalance cost produced by a group of renewable energy firms that bid individually in the day ahead market but decide to cooperate by sharing the imbalance costs that they produce. The proposed cost allocation mechanism has been validated using real data for an aggregation of five wind farms. The new framework has interest for both balancing authorities and aggregator companies that manage a large number of individual producers. By correctly assigning the costs to those that generated them, transparent signals are sent to the market agents that encourage economic efficiency and promote a fair renewable energy integration. We plan to study the application of this framework to demand side management (DSM) problems, where an incentive has to be shared among a large number of consumers that cooperate to reduce power consumption.

REFERENCES

[1] H. Holttinen *et al.*, “Impacts of large amounts of wind power on design and operation of power systems, results of IEA collaboration,” *Wind Energy*, vol. 14, no. 2, pp. 179–192, 2011.
[2] MIT Energy Initiative, “Integration of variable energy resources,” in *The Future of the Electric Grid*. Cambridge, MA, USA: Massachusetts Inst. Technol., 2011, ch. 3.
[3] M. Madrigal and K. Porter, *Operating and Planning Electricity Grids With Variable Renewable Generation: Review of Emerging Lessons From Selected Operational Experiences and Desktop Studies*. World Bank Publications, 2013.

[4] A. Helander, H. Holttinen, and J. Paatero, “Impact of wind power on the power system imbalances in Finland,” *IET Renew. Power Gener.*, vol. 4, no. 1, pp. 75–84, 2010.
[5] H. Holttinen *et al.*, “Impacts of large amounts of wind power on design and operation of power systems, results of IEA collaboration,” *Wind Energy*, vol. 14, no. 2, pp. 179–192, 2011.
[6] J. C. Smith, M. R. Milligan, E. A. DeMeo, and B. Parsons, “Utility wind integration and operating impact state of the art,” *IEEE Trans. Power Syst.*, vol. 22, no. 3, pp. 900–908, Aug. 2007.
[7] U.S. Energy Information Administration, “Preliminary monthly electric generator inventory,” 2015. [Online]. Available: https://www.eia.gov/todayinenergy/detail.php?id=25492#
[8] C. Brunetto and G. Tina, “Wind generation imbalances penalties in day-ahead energy markets: The Italian case,” *Electr. Power Syst. Res.*, vol. 81, no. 7, pp. 1446–1455, 2011.
[9] K. Porter, K. Starr, and A. Mills, “Variable generation and electricity markets. A living summary of markets and market rules for variable generation in North America,” Utility Variable Gener. Integr. Group, Reston, VA, USA, Tech. Rep., 2015.
[10] EnerNex Corp., “Eastern wind integration and transmission study,” Nat. Renew. Energy Lab., Golden, CO, USA, Tech. Rep. NREL/SR-5500-47078, Feb. 2011.
[11] G. Maser, “It’s electric, but FERC’s cost-causation Boogie-Woogie fails to justify socialized costs for renewable transmission,” *Georgetown Law J.*, vol. 100, pp. 1829–1854, 2011.
[12] W. W. Hogan, “Revenue sufficiency guarantees, cost causation, and cost allocation,” Comments submitted to the Federal Energy Regulatory Commission, Washington, DC, USA, Docket no. EL07-86-000, 2008.
[13] P. M. Sotkiewicz and J. M. Vignolo, “Towards a cost causation-based tariff for distribution networks with DG,” *IEEE Trans. Power Syst.*, vol. 22, no. 3, pp. 1051–1060, Aug. 2007.
[14] J. C. Bonbright, *Principles of Public Utility Rates*. New York, NY, USA: Columbia Univ. Press, 1961.
[15] B. Kirby, M. Milligan, and Y. Wan, “Cost-causation-based tariffs for wind ancillary service impacts,” in *Proc. Windpower*, 2006.
[16] E. Baeyens, E. Bitar, P. P. Khargonekar, and K. Poolla, “Wind energy aggregation: A coalitional game approach,” in *Proc. 50th IEEE Conf. Decision Control/Eur. Control Conf.*, 2011, pp. 3000–3007.
[17] E. Baeyens, E. Y. Bitar, P. P. Khargonekar, and K. Poolla, “Coalitional aggregation of wind power,” *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3774–3784, Nov. 2013.
[18] W. Lin and E. Bitar, “Forward electricity markets with uncertain supply: Cost sharing and efficiency loss,” in *Proc. IEEE 53rd Annu. Conf. Decision Control*, 2014, pp. 1707–1713.
[19] A. Nayyar, K. Poolla, and P. Varaiya, “A statistically robust payment sharing mechanism for an aggregate of renewable energy producers,” in *Proc. Eur. Control Conf.*, 2013, pp. 3025–3031.
[20] P. Chakraborty, E. Baeyens, P. P. Khargonekar, and K. Poolla, “A cooperative game for the realized profit of an aggregation of renewable energy producers,” in *Proc. 55th IEEE Conf. Decision Control*, 2016, pp. 5805–5812.
[21] H. P. Young, Ed., *Cost Allocation: Methods, Principles, Applications*. Amsterdam, The Netherlands: North Holland, 1985.
[22] H. Moulin, “Axiomatic cost and surplus sharing,” in *Handbook of Social Choice and Welfare*, vol. 1. New York, NY, USA: Elsevier, 2002, pp. 289–357.
[23] J. L. Hougaard, *An Introduction to Allocation Rules*. Berlin, Germany: Springer, 2009.
[24] J. Von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*. Princeton, NJ, USA: Princeton Univ. Press, 1944.
[25] M. J. Osborne and A. Rubinstein, *A Course in Game Theory*. Cambridge, MA, USA: MIT Press, 1994.
[26] L. S. Shapley, “On balanced sets and cores,” *Naval Res. Logistics Quart.*, vol. 14, no. 4, pp. 453–460, 1967.
[27] O. N. Bondareva, “Some applications of linear programming methods to the theory of cooperative games,” *Probl. Kibernetiki*, vol. 10, pp. 119–139, 1963.
[28] M. Milligan *et al.*, “Integration of variable generation, cost-causation, and integration costs,” *Electricity J.*, vol. 24, no. 9, pp. 51–63, 2011.
[29] E. Friedman and H. Moulin, “Three methods to share joint costs or surplus,” *J. Econ. Theory*, vol. 87, no. 2, pp. 275–312, 1999.
[30] E. Y. Bitar, R. Rajagopal, P. P. Khargonekar, K. Poolla, and P. Varaiya, “Bringing wind energy to market,” *IEEE Trans. Power Syst.*, vol. 27, no. 3, pp. 1225–1235, Aug. 2012.
[31] Red Eléctrica de España (REE), “Sistema de información del operador del sistema (esios),” 2016. [Online]. Available: https://www.esios.ree.es/en
[32] WU, “Weather Underground website,” 2016. [Online]. Available: https://www.esios.ree.es/en
[33] Vestas Wind Sytems A/S, “V52-850 kW. The turbine that goes everywhere,” 2016. [Online]. Available: https://www.vestas.com

Pratyush Chakraborty (M’17) received the B.E. degree in electrical engineering from Jadavpur University, Kolkata, India, in 2006, the M.Tech. degree in electrical engineering from the Indian Institute of Technology, Bombay, Mumbai, India, in 2011, and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of Florida, Gainesville, FL, USA, in 2013 and 2016, respectively. From 2006 to 2009, he worked as a Graduate Trainee Engineer for the initial one year, as a Marketing Executive for the next 1.5 years, and as a Senior Marketing Executive for the next six months in the Industrial Solutions and Services Division, Siemens Limited, Kolkata, India. He is currently a Postdoctoral Scholar with the Department of Mechanical Engineering, University of California, Berkeley, CA, USA. His research interests include game theory, optimization and control, and their applications to electrical power systems.

Enrique Baeyens (M’95) received the Industrial Engineering and Ph.D. degrees from the University of Valladolid, Valladolid, Spain, in 1989 and 1994, respectively. In 1997, he joined the University of Valladolid, where he is currently a Professor with the Department of Systems Engineering. From 1999 to 2002, he was the Associate Dean for Research at the College of Engineering, University of Valladolid. In 2007, he became the Director of Research of CARTIF, a Spanish application-oriented research organization, which develops technological innovations and novel systems solutions for industrial customers and especially for SMEs. He also served as the Director of the Instituto de las Tecnologías Avanzadas de la Producción from 2011 to 2015. His research interests include optimal control theory, modeling and system identification, and its applications to industrial and power systems.

Pramod P. Khargonekar (F’93) received the B.Tech. degree in electrical engineering from the Indian Institute of Technology, Bombay, Mumbai, India, in 1977, and the M.S. degree in mathematics and the Ph.D. degree in electrical engineering from the University of Florida, Gainesville, FL, USA, in 1980 and 1981, respectively. After holding faculty positions in electrical engineering at the University of Florida and the University of Minnesota, he joined the University of Michigan in 1989 as a Professor of electrical engineering and computer science. He became the Chairman of the Department of Electrical Engineering and Computer Science in 1997 and also held the position of Claude E. Shannon Professor of Engineering Science. He was the Dean of Engineering from 2001 to 2009 and Eckis Professor of Electrical and Computer Engineering from 2001 to 2016 at the University of Florida. He served briefly as the Deputy Director of Technology at ARPA-E, U.S. Department of Energy in 2012–2013. He was the Assistant Director for the Directorate of Engineering at the National Science Foundation (NSF) from 2013 to 2016. In June 2016, he assumed his current position as the Vice Chancellor for Research and a Distinguished Professor of electrical engineering and computer science at the University of California, Irvine, CA, USA. He received the NSF Presidential Young Investigator Award, the American Automatic Control Council’s Donald Eckman Award, the Japan Society for Promotion of Science fellowships, the IEEE W. R. G. Baker Prize Award, the IEEE CSS George Axelby Best Paper Award, the Hugo Schuck ACC Best Paper Award, and the Distinguished Alumnus and Distinguished Service Awards from the Indian Institute of Technology, Bombay. He is a Fellow of the IFAC.