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Flexible Loads and Renewable Integration: Distributed Control and Price of Anarchy*

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***Abstract*— New sources of uncertainty and variability are being introduced into modern power grids creating new control challenges. Examples include renewable generation from solar and wind generators, electric vehicles, etc. In addition, there is compelling value in reducing the peak electric power demand as that has a direct beneficial impact of reducing the need for new capital investments in overall power sector. Introduction of new sensing, communications and computational elements offers opportunities for novel control solutions. One promising approach to addressing these problems is to exploit the *inherent flexibility* in many types of electric power loads and use that to accommodate the inherent variability in renewable generation and/or to reduce the peak demand. In this paper, we focus on electric vehicles(EVs) as flexible loads in the context of renewable generation. We take an intra-day time horizon where we assume we have a good prediction of renewable generation. Based on the supply schedule of thermal generators and predicted supply of renewable generation, the charging of the electric vehicles is controlled to minimize the imbalance between generation and consumption using centralized and distributed control algorithms. We develop a pricing scheme based on the *proportional allocation mechanism* for the distributed case. Assuming individual loads are *price takers,* we show that there is a time varying price which can be set by the control authority such that it's objective aligns with the individual's objective. If the users are *price anticipators,* the corresponding situation can be formulated in a game-theoretic setting. Distributed algorithms are developed to compute solution in both the cases. We also analyze the "price of anarchy" and show that the worst case loss of efficiency is 0.25.**

I. INTRODUCTION

Concerns over carbon emissions, climate change, and sustainability are motivating a large global effort to integrate renewable electric energy sources such as wind and solar into the power grid. These sources of electric power are inherently uncertain, variable and non-dispatchable. They are uncertain in the sense that the generated power is a stochastic process as it depends on wind speed or solar insolation. Even if one could predict them perfectly, they would still vary with time unlike traditional gas, coal, or nuclear generators. Finally, non-dispatchability refers to the fact that their output cannot be controlled to follow an external command, again in contrast with traditional sources of electric power. The term *variability* is used to capture these three characteristics [1]. Variability of renewable generation is a major challenge in power systems operations, particularly when their contribution to the total generation becomes large [2].

Peak power demand is the main factor that drives capital investment decisions in generation, transmission and distribution infrastructure. Economic growth, population growth, and new electric energy consumption devices such as electric vehicles and plug-in hybrid electric vehicles influence peak power demand. A cursory examination of the load duration curve shows that a large fraction of the generation capacity goes unused most of the time as the peak demand occurs in a relatively small number of hours of the year [3].

A major constraint in power systems operation is that generation and consumption must remain in balance on an instant-by-instant basis [4]. [This is a consequence of the fact that grid scale energy storage is essentially non-existent although this could change in the coming years as a tremendous amount of research and development efforts are currently underway in this direction.] Traditionally, power demand is assumed to be uncontrolled and (controllable or dispatchable) supply is adjusted using a sophisticated scheme of feed-forward control (using day-ahead open markets and real time markets) and feedback control (frequency regulation) so that the supply and demand are equal. One promising approach to addressing the problems of renewable integration and peak demand reduction is to exploit the inherent flexibility of some of the loads such as water heaters, washers, dryers, refrigerators, heating, air-conditioning, etc. Electric vehicles are an excellent example of flexible loads. Impact of EVs in electrical power demand side management has been studied in detail in [5], [6]. They can be very helpful in absorbing the variability of renewable energy at the system level [7], [8]. While we are using EVs as the main example of flexible loads in this paper, the ideas and results of the paper also apply to other types of flexible loads.

Taking advantage of flexibility of EVs and other flexible loads and imposing suitable time-varying prices, the total cost of energy consumption can be reduced as studied in [9] and [10]. But these papers do not consider variable supply sources and utility functions of the loads. In [10], different approximations are made considering very large population of electric vehicles and the Nash equilibrium is shown to be optimal when the charging rates of all the vehicles are same, which may be a restrictive assumption. Game theoretic behavior when flexible loads are forced to bid in multiperiod day ahead power markets under uniform price quantity bidding rules has been studied in [11].

Addressing wind variability, a game is formulated among various power consumers in [12], and the price of anarchy

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is calculated for an example problem. In [13], applying time varying prices on demand side, supply of generators is scheduled efficiently. But with significant wind and solar in the generation mix, this has to be done even intra-day (say 8 hrs ahead, 4 hrs ahead) to reduce the stress on real-time market, which will incur additional cost of running and operating a new market.

This paper is focused on EV flexible loads and renewable integration. We assume there is a traditional day ahead market, where based on demand predictions, the supply is scheduled. During the operating day, the central authority gets a better forecast of renewable generation. Based on this forecast, it sets electric energy price by applying a "proportional allocation mechanism". This is a new mechanism which has not been previously examined in the setting of electric grid with renewables. It has the potential to contribute to the integration of renewables and harnessing load flexibility. However, how this can be harmonized with electricity market mechanisms remains to be studied. The proportional allocation mechanism was first introduced by Kelly in [14] and Kelly et al. in [15] in a communication network problem. Some recent works, [16], [17], have explored the proportional allocation mechanism to control EV charging. In [16], a distributed charging control problem has been formulated and simulation results are obtained. The charging of EVs is coordinated using a policy inspired by lottery scheduling in [17].

In this paper, we model electric vehicles with operational constraints and utility functions. We then formulate and solve a centralized control problem to manage the flexible loads. Assuming loads are price takers, we formulate a distributed control problem where the price is set up by applying proportional allocation mechanism resulting in a competitive equilibrium. An algorithm is developed to compute the equilibrium. For the case of price anticipators, we formulate and solve a game and show that the Nash equilibrium exists and is unique. An algorithm is developed to compute the Nash equilibrium. It is intuitive that centralized control will achieve better performance than distributed control.

The concept of "price of anarchy" is used to quantify the loss of efficiency in using distributed control in the *price anticipatory* case. Price of anarchy has become a topic of great research interest in the communication and computer science literature [18], [19], [20], [21]. In [18] and [19] the authors derived bounds on the price of anarchy for various types of games with cost sharing. In [20], a tight bound for routing games has been developed which is called Pigou bound. Johari, in [21] has shown a tight bound on the price of anarchy where a single infinitely divisible resource has been allocated among multiple competing users. In [22] it has been shown that non-cooperative behavior of self-interested agents leads to suboptimal outcome as compared to co-operative behavior of agents in dynamic oligopoly market structure. But no concrete bound on price of anarchy has been derived. We analyze the price of anarchy for our case of flexible loads and renewable integration and show that, the loss of efficiency is 25% in the worst case. To the best of our knowledge, this is the first analytical result on price of anarchy in the smart power grids context.

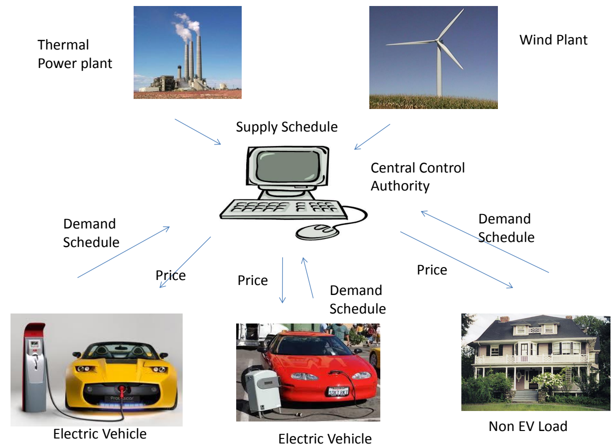

Fig. 1. System Model

II. Problem Formulation

Consider a system with thermal and renewable generators, uncontrollable loads, and flexible loads consisting of N electric vehicles whose charging schedules are flexible. The time horizon T is discretized as $(1,2,\ldots,T)$ and indexed by t. (We envision an intra-day time horizon T of 8 hours and our method is applied 2-4 hour ahead of actual power consumption) Each EV user possesses a smart meter with advanced two way communication capability.

A. Generation and Loads

Let $d_i(t)$ be the power consumption of the i-th EV at time t. Let $c(t)$ be the total scheduled power generation of all the thermal power plants at time t. This is envisioned to be obtained from the settlement of day-ahead market along with some intra-day adjustments. Let $w(t)$ be the total predicted power supply of the renewable generators at time t. Let $n(t)$ denote total power consumption of all uncontrolled loads, i.e., all loads other than the flexible (EV) loads, at time t. Estimate of $n(t)$ can be obtained either from load forecasting algorithms or (in the future) via some smart grid based approaches. The objective of our problem is to balance supply with demand i.e.

$$c(t)+w(t)=n(t)+\sum_{i=1}^{N} d_i(t) \quad \forall t \in T \tag{1}$$

Let us define

$$v(t):=c(t)+w(t)-n(t). \tag{2}$$

Now we assume that

$$v(t)>0 \tag{3}$$

for all t. This means that the scheduled thermal generation and renewable production are sufficient to meet the uncontrollable demand. The supply demand balancing is done by adjusting the charging of the EVs. We do not allow vehicle to grid power transfer here. i.e.

$$d_i(t) \geq 0 \tag{4}$$

for all i and t. We note that there will be inevitable mismatch between forecasts of $w(t), n(t)$ and their actual values in real-time. These errors will be handled by ancillary services such as frequency regulation or load following. The idea is that by using flexible loads to satisfy (1), we will reduce the need for these ancillary services and make large scale integration of renewable generation less burdensome.

B. EV Modeling

Let U be the utility function of the EVs in monetary units. The Utility function U is assumed to be a concave, strictly increasing and continuously differentiable function. For flexible loads, the utility function is inter-temporal [23], [11]. The overall utility across time T is a function of the total amount of energy consumed during the time T, i.e.,

$$\sum_{t=1}^{T} U(d_i(t)) = U(\sum_{t=1}^{T} d_i(t)) \tag{5}$$

The flexibility of EV charging loads $d_i(t)$ is constrained by the following requirements:

- The charging rate is limited by d_i^{max} and d_i^{min}:
$$d_i^{min} \leq d_i(t) \leq d_i^{max} \tag{6}$$
where d_i^{min} is nonnegative as per (4).
- The total energy consumption of the electric vehicle is bounded below and above by q_i^{min} and q_i^{max}:
$$q_i^{max} \geq \sum_{t=1}^{T} d_i(t) \geq q_i^{min} \tag{7}$$
.

These constraints are linear. So we can write them together in the form

$$\sum_{t=1}^{T} \gamma_i^m(t) d_i(t) \leq b^m \quad \forall m \in M, \, i \in N \tag{8}$$

where M denotes the total number of constraints.

III. Centralized Control

We first consider the case where there is a central control authority that aims to maximize the total utility of consumers while satisfying power balance constraint at each time. This can be regarded as a theoretical ideal case against which distributed control solutions can be compared. Thus, the control authority's objective is to maximize V, where V is given by

$$V = \sum_{i=1}^{N} \sum_{t=1}^{T} U(d_i(t)) \tag{9}$$

subject to equations (1) and (8).

A1: We assume that the convex set produced by the set of constraints defined by equations (1) and (8) is nonempty.

A2: We also assume that all the optimization problems in this paper are solved by gradient descent method.

Since we will be dealing with concave optimization problems with convex constraints, the first assumption will ensure that a global maximum exists and is unique. It can then be found using the well-known KKT conditions [24]. In order to calculate these KKT conditions, let us define

$$\tilde{V} = \sum_{i=1}^{N} \sum_{t=1}^{T} U(d_i(t)) - \sum_{t=1}^{T} \lambda(t)(n(t) + \sum_{i=1}^{N} d_i(t)$$
$$-c(t) - w(t)) - \sum_{i=1}^{N} \sum_{m=1}^{M} \mu_m^i (\sum_{t=1}^{T} \gamma_i^m(t) d_i(t) - b^m) \tag{10}$$

where $\lambda(t)$ and μ_m^i are the Lagrange multipliers. Now taking partial derivatives with respect to $d_i(t)$ and $\lambda(t)$ and writing the complementary slackness condition for the inequality constraints, we get the following KKT conditions:

$$U'(d_i(t)) - \lambda(t) - \sum_{m=1}^{M} \mu_m^i \gamma_i^m(t) = 0 \tag{11}$$

$$c(t) + w(t) = n(t) + \sum_{i=1}^{N} d_i(t) \tag{12}$$

$$\mu_m^i (\sum_{t=1}^{T} \gamma_i^m(t) d_i(t) - b^m) = 0 \tag{13}$$

$$\mu_m^i \geq 0 \tag{14}$$

for all $i \in N$, $t \in T$, $m \in M$.

If the central control authority has complete knowledge of all the EV users' utility functions and constraints, then in principle, it can compute the optimal solution by solving the above equations. We will refer to this idealized scenario as the centralized optimal solution and denote it by $d_i^s(t)$.

IV. Distributed Control with Price Taking EV Users

For the centralized control case discussed above, the central authority has to know utility functions and operational constraints of all the vehicles at all times. The EV users may not want to disclose their utility functions and constraints. The users may also want to control their EV charging independently. Also, the dimension of the optimization problem could be very large considering large numbers of EV users (or flexible loads). Therefore, we next propose a distributed control policy.

In this section, we will assume that the EV users are price takers. [The case of price anticipating users will be considered in the next section.] We also consider them as selfish and rational. Let $k_i(t)$ denote the monetary value associated with consumption $d_i(t)$. At first the control authority, after obtaining the values of $c(t)$, $w(t)$ and $n(t)$, calculates $v(t)$ according to (2) and broadcasts the value to all the users. Each user then submits its $k_i(t)$ to the central control authority for all t. The central control authority calculates $\sum_{i=1}^{N} k_i(t)$ for each time t and decides price according to the following formula

$$p(t) = \frac{\sum_{i=1}^{N} k_i(t)}{v(t)} \tag{15}$$

where $v(t) \neq 0$ as per (3). This is inspired by proportional allocation mechanism where the allocation of $d_i(t)$ to the i-th user is given by

$$d_i(t) = \frac{k_i(t)}{p(t)} \tag{16}$$

for all i and t. The control authority does not price discriminate according to users. So, each vehicle is charged the same price $p(t)$. All the uncontrollable loads are also charged the same price though their consumption does not depend on the change in price. Now as the users are selfish and rational, each user i will try to maximize its own utility function and minimize its cost of energy consumption by a suitable selection of its strategy $k_i(t)$. Thus, each user i will try to maximize L_i

$$L_i = \sum_{t=1}^{T} U(\frac{k_i(t)}{p(t)}) - \sum_{t=1}^{T} k_i(t) \tag{17}$$

subject to the constraints

$$\sum_{t=1}^{T} \gamma_i^m(t) \frac{k_i(t)}{p(t)} \leq b^m. \tag{18}$$

We now define i-th user's strategy as $k_i = (k_i(t))$ and price in the network as $p = (p(t))$ for all t. We say that (k_i^*, p^*) is a *competitive equilibrium* if users maximize their payoff defined as (17) and the central control authority clears the market by setting the price according to (15). Mathematically,

$$L_i(k_i^*, p^*) \geq L_i(\hat{k}_i, \hat{p}) \; \forall \, i \in N \tag{19}$$

where $\hat{k}_i$ and $\hat{p}$ are all possible consumption schedules.
The distributed control problem is to find a collection of user strategies $k_i(t)$ and price $p(t)$ resulting in a competitive equilibrium.

Theorem 4.1: Consider the distributed control problem where objective function is defined by (17) and constraints are defined by (18). Assuming that the convex set produced by the set of constraints is nonempty, a competitive equilibrium solution exists and is unique.

Proof: Now as the objective function is concave and constraints are convex, the global maximum for the i-th user exists and is unique. The maximum can be found from the KKT conditions. In order to find the KKT conditions, let us define

$$\begin{aligned}\tilde{L}_i = &\sum_{t=1}^{T} U(\frac{k_i(t)}{p(t)}) - \sum_{t=1}^{T} k_i(t) - \\ &\sum_{m=1}^{M} \mu_m^i (\sum_{t=1}^{T} \gamma_i^m(t) \frac{k_i(t)}{p(t)} - b^m)\end{aligned} \tag{20}$$

where μ_m^i is the Lagrange multiplier. Now taking partial derivatives with respect to $k_i(t)$ and writing the complementary slackness condition for the inequality constraints, we get the following KKT conditions:

$$U'(\frac{k_i(t)}{p(t)}) - p(t) - \sum_{m=1}^{M} \mu_m^i \gamma_i^m(t) = 0 \tag{21}$$

$$\mu_m^i (\gamma_i^m(t) \frac{k_i(t)}{p(t)} - b^m) = 0 \tag{22}$$

$$\mu_m^i \geq 0 \tag{23}$$

for $t \in T$, $m \in M$.

Now if we compare equations (21) -(23) for all i with that of (11)-(14), we see that the centralized and distributed problem have the same solution iff $c(t) + w(t) = n(t) + \frac{\sum_i k_i(t)}{p(t)}$ where $d_i(t) = \frac{k_i(t)}{p(t)}$. The price $p(t)$ is actually the Lagrange multiplier of the centralized problem. So a competitive equilibrium of the system exists and is unique. ∎

A. Algorithm to Compute the Price and Consumption Schedule

Next we develop a distributed algorithm, where the central control authority and EV users jointly compute the competitive equilibrium consumption schedule and the price .

1) The control authority has the information of $c(t)$, $w(t)$ and $n(t)$ for all t. It computes the value of $v(t)$ as per (2) for all t and broadcasts to all the users.
2) Each user i sends his $k_i(t)$ for all t to the control authority.
3) The control authority adds all the $k_i(t)$s to obtain $\sum_i k_i(t)$. It then computes $p(t)$ according to (15) and broadcasts to all the users.
4) The users solve their own optimization problems applying the KKT conditions (21)-(23) and get the new $k_i(t)$s for all t.
5) The process from step (2)-step (4) is continued until convergence is achieved. It can be shown that the algorithm will converge if the step lengths in the gradient descent methods of each optimization problem are sufficiently small [25]. In this paper, we have not included an analysis of the speed of convergence of our algorithm.

V. Distributed Control with Price Anticipating EV Users

Above analysis has been carried out assuming that EV users are price takers. If users are price anticipators, they will try to account for the impact of their decisions on $p(t)$ and adjust their decisions accordingly. Suppose they know that $p(t)$ is decided by the formula $p(t) = \frac{\sum_{i=1}^{N} k_i(t)}{v(t)}$. We model the resulting situation as a noncooperative game. The game of energy consumption is as follows:

1) Players: Set of N end users
2) Strategy: user i's strategy $k_i = (k_i(t))$ for all $t \in T$
3) Payoff: For each user i, the payoff is equal to

$$L_i(k_i, k_{-i}) = \sum_{t=1}^{T} U(\frac{k_i(t)v(t)}{\sum_i k_i(t)}) - \sum_{t=1}^{T} k_i(t) \tag{24}$$

subject to

$$\sum_{t=1}^{T} \gamma_i^m(t) \frac{k_i(t)v(t)}{\sum_i k_i(t)} \leq b^m \tag{25}$$

where k_{-i} is the power consumption of all users other than the user i. Each user will try to maximize his own payoff assuming all other users' strategies are fixed. This is called the "best response strategy". We define Nash equilibrium which is a set of all players' strategies such that no player has an incentive to deviate unilaterally. Mathematically, Nash equilibrium is the strategy k_i^* such that,

$$L_i(k_i^*, k_{-i}^*) \geq L_i(k_i, k_{-i}^*) \ \forall i \in N. \tag{26}$$

Now let us take

$$\tilde{L}_i = \sum_{t=1}^{T} U(\frac{k_i(t)v(t)}{\sum_{i=1}^{N} k_i(t)}) - \sum_{t=1}^{T} k_i(t) - \sum_{m=1}^{M} \mu_m^i (\sum_{t=1}^{T} \gamma_i^m(t) \frac{k_i(t)v(t)}{\sum_{i=1}^{N} k_i(t)} - b^m) \tag{27}$$

where μ_m^i is the Lagrange multiplier. The conditions for the Nash equilibrium are

$$(U'(\frac{k_i(t)v(t)}{\sum_{i=1}^{N} k_i(t)}) - \sum_{m=1}^{M} \mu_m^i \gamma_i^m(t))(1 - \frac{k_i(t)}{\sum_{i=1}^{N} k_i(t)}) = \frac{\sum_{i=1}^{N} k_i(t)}{v(t)} \tag{28}$$

$$\mu_m^i (\gamma_i^m(t) \frac{k_i(t)}{p(t)} - b^m) = 0 \tag{29}$$

$$\mu_m^i \geq 0 \tag{30}$$

for $t \in T, m \in M$.

Now, we will show that the Nash equilibrium solution of the game exists and is unique.

Theorem 5.1: Consider the energy consumption game where objective function is defined by (24) and constraints are defined by (25). Assuming the convex set produced by the constraints is non empty, a Nash equilibrium solution exists and is unique.

Proof: We consider an optimization problem of maximizing Q, where Q is given by the following equation

$$Q = \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{U}(d_i(t)) \tag{31}$$

subject to equations (1) and (8). Here $\hat{U}(d_i(t))$ is defined as

$$\hat{U}(d_i(t)) = (1 - \frac{d_i(t)}{v(t)})u(d_i(t)) + \frac{1}{v(t)} \int_0^{d_i(t)} U(z)dz - \sum_{m=1}^{M} \mu_m^i \gamma_i^m(t)(d_i(t) - \frac{d_i^2(t)}{2v(t)}) \tag{32}$$

As this is a concave optimization problem with convex constraints, the global maxima exists and is unique. The maxima can be found from the KKT conditions. In order to find the KKT conditions, we start with

$$\tilde{Q} = \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{U}(d_i(t)) - \sum_{t=1}^{T} \lambda(t)(n(t) + \sum_{i=1}^{N} d_i(t) - c(t) - w(t)) - \sum_{i=1}^{N} \sum_{m=1}^{M} \mu_m^i (\sum_{t=1}^{T} \gamma_i^m(t) d_i(t) - b^m) \tag{33}$$

where $\lambda(t)$ and μ_m^i are Lagrange multipliers. Now taking partial derivatives with respect to $d_i(t)$ and $\lambda(t)$ and writing the complementary slackness condition for the inequality constraints, we get the following KKT conditions:

$$(U'(d_i(t)) - \sum_{m=1}^{M} \mu_m^i \gamma_i^m(t))(1 - \frac{d_i(t)}{v(t)}) = \lambda(t) + \sum_{m=1}^{M} \mu_m^i \gamma_i^m(t) \tag{34}$$

$$c(t) + w(t) = n(t) + \sum_{i=1}^{N} d_i(t) \tag{35}$$

$$\mu_m^i (\gamma_i^m(t) d_i(t) - b^m) = 0 \tag{36}$$

$$\mu_m^i \geq 0. \tag{37}$$

for all $i \in N, t \in T, m \in M$.

Now these equations are exactly same as the equations (28)-(30) with

$$\lambda(t) = \frac{\sum_{i=1}^{N} k_i(t)}{v(t)} - \sum_{m=1}^{M} \mu_m^i \gamma_i^m(t) \tag{38}$$

where $d_i(t) = \frac{k_i(t)v(t)}{\sum_{i=1}^{N} k_i(t)}$.

So the Nash equilibrium solution is the optimal solution to the above optimization problem. As the optimization problem has an unique maxima, so does the game. ■

We denote the Nash equilibrium solution by $d_i^g(t)$ while comparing with centralized optimal solution $d_i^s(t)$ in section VI.

A. Algorithm to Compute The Nash Equilibrium Consumption Schedule

Next we develop a distributed algorithm, where the control authority and EV users jointly compute the Nash equilibrium.

1) The control authority has the information of $c(t)$, $w(t)$ and $n(t)$ for all t. It computes the value of $v(t)$ as per (2) for all t and broadcasts to all the users.
2) Each user i sends his $k_i(t)$ for all t to the authority.
3) The authority adds all the $k_i(t)$s to obtain $\sum_i k_i(t)$ and computes $p(t)$ according to equation (15) for all t and broadcasts to all the users.
4) The users compute k_{-i} as they know the formula for $p(t)$ and solve their optimization problems applying the KKT conditions (28)-(30) and get new $k_i(t)$s for all t.
5) The process (2)-(4) is continued until convergence is achieved. It can be shown that the algorithm will converge if the step lengths of the gradient descent methods in each optimization problem are sufficiently

small [25]. Like the price taker case, here also we have not included an analysis of the speed of convergence of our algorithm.

VI. Price of Anarchy

It is intuitive that the optimal centralized control will achieve better performance than distributed control in the noncooperative price anticipation game. Price of anarchy is a concept to quantify the loss of efficiency in using distributed control over centralized control. The price of anarchy is defined as the worst-case ratio of the objective function value of a Nash equilibrium of a game and that of a centralized optimal solution. It quantifies the inefficiency of selfish behavior [19] as compared with optimal centralized control.

Theorem 6.1: Consider the centralized optimal solution $d_i^s(t)$ and the distributed Nash equilibrium solution $d_i^g(t)$. Let F be defined by

$$F := \frac{\sum_{i=1}^N \sum_{t=1}^T U(d_i^g(t))}{\sum_{i=1}^N \sum_{t=1}^T U(d_i^s(t))}. \tag{39}$$

Then $F \geq 0.75$ and the bound is tight.

Proof: Putting $d_i(t) = \frac{k_i(t)v(t)}{\sum_{i=1}^N k_i(t)}$ in the equation (28), we get

$$(U'(d_i(t)) - \sum_{m=1}^M \mu_m^i \gamma_i^m(t))(1 - \frac{d_i(t)}{v(t)}) = \frac{\sum_{i=1}^N k_i(t)}{v(t)}. \tag{40}$$

Now as

$$\hat{U}'(d_i(t)) = (U'(d_i(t)) - \sum_{m=1}^M \mu_m^i \gamma_i^m(t))(1 - \frac{d_i(t)}{v(t)}) \tag{41}$$

we can write

$$\hat{U}'(d_i(t)) = \frac{\sum_{i=1}^N k_i(t)}{v(t)} = \zeta(t)(say). \tag{42}$$

Let us take a Nash equilibrium solution $d_i(t)$ and another solution $\hat{d}_i(t)$ satisfying the supply-demand balance equation (1). Therefore,

$$\sum_{i=1}^N (d_i(t) - \hat{d}_i(t)) = 0. \tag{43}$$

So, combining equations (42) and (43),we get

$$\sum_{t=1}^T \sum_{i=1}^N \hat{U}'(d_i(t))(d_i(t) - \hat{d}_i(t)) = \sum_{t=1}^T \zeta(t) \sum_{i=1}^N (d_i(t) - \hat{d}_i(t)) = 0. \tag{44}$$

As the equation (44) satisfies the condition of corollary 21.3 of [21] and we assumed $U(0) \geq 0$, according to the theorem 21.4 of [21]

$$\sum_{i=1}^N \sum_{t=1}^T U(d_i^g(t)) \geq \frac{3}{4} \sum_{i=1}^N \sum_{t=1}^T U(d_i^s(t)) \tag{45}$$

and furthermore, this bound is tight: for every $\varepsilon > 0$, there exists a choice of N, and T and a choice of (linear) utility functions U such that,

$$\sum_{i=1}^N \sum_{t=1}^T U(d_i^g(t)) \leq (\frac{3}{4} + \varepsilon) \sum_{i=1}^N \sum_{t=1}^T U(d_i^s(t)). \tag{46}$$

Thus the bound on F of 0.75 is tight and the worst case efficiency loss is 25 percent. ■

VII. Conclusion and Future Work

In this paper, we have investigated supply-demand balancing for renewable integration using flexible loads. We explored centralized and distributed solutions. By a distributed control policy, the supply of constrained electric energy production is allocated to the EV users. We show that a time varying price can be found such that with price taking EV users, their individual objective function aligns with central control authority's objective. If this method is applied to a power distribution system with high renewable penetration in an intra-day time horizon, it can help in mitigating supply-demand imbalance. We also investigated the more interesting case of price anticipating consumers and showed that a Nash equilibrium exists. We also obtained a tight bound for the worst case price of anarchy.

We are currently exploring several questions in this general direction:

1) How to model and analyze analogous problems for flexible thermal loads? There will be a need to account for more complicated load dynamics.
2) How to handle the problem when supply is less than non EV demand and vehicle to grid energy transfer is allowed?
3) The methods presented here will be applied offline in a system based on the predicted supply of renewable generation. There will always be some prediction error, even if the prediction horizon is short. The resulting imbalance will be handled by ancillary services such as regulation and load following. We are exploring methods to include prediction errors in our problem formulation and analyzing the impact on regulation and load following needs.

References

[1] NERC Special Report, "Accomodation of high levels of variable generation," Tech. Rep., april 2009.

[2] E. Bitar, P. Khargonekar, and K. Poolla, "Systems and control opportunities in the integration of renewable energy into the smart grid," in *Proceedings of the 18th IFAC World Congress*, Milano (Italy), Aug-Sep 2011, pp. 4927–4932.

[3] P. Denholm and W. Short, "An evaluation of utility system impacts and benefits of optimally dispatched plug-in hybrid electric vehicles," National Renewable Energy Laboratory, Amherst, MA, Technical Report NREL/TP-620-40293, 2006.

[4] A. R. Bergen and V. Vittal, *Power Systems Analysis*. Upper Saddle River, NJ: Prentice Hall, 1986.

[5] Electrification Coalition, "Electrification roadmap: Revolutionizing transportation and achieving energy security," Tech. Rep., Nov. 2009.

[6] S. Rehman and G. Shrestha, "An investigation into the impact of electric vehicle load on the electric utility distribution system," *IEEE Transactions on Power Delivery*, vol. 8, no. 2, pp. 591–597, 1993.

[7] W. Kempton and J. Tomic, "Vehicle-to-grid power fundamentals: Calculating capacity and net revenue," *Journal of Power Sources*, vol. 144, no. 1, pp. 268–279, 2005.
[8] ——, "Vehicle-to-grid power implementation: From stabilizing the grid to supporting large scale renewable energy," *Journal of Power Sources*, vol. 144, no. 1, pp. 280–294, 2005.
[9] A. Mohsenian-Rad, V. Wong, J. Jatskevich, R. Schober, and A. L. Garcia, "Autonomous demand-side management based on game-theoritic energy consumption scheduling for the future smart grid," *IEEE Transactions on Smart Grid*, vol. 1, no. 3, pp. 320–331, 2010.
[10] Z. Ma, D. Callaway, and I. Hiskens, "Decentralized charging control of large populations of plug-in electric vehicles," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 1, pp. 67–78, 2013.
[11] M. C. Caramanis and J. Foster, "Uniform and complex bids for demand response and wind generation scheduling in multi-period linked transmission and distribution markets," in *50th IEEE Conference on Decision and Control*, Orlando, FL, Dec. 2011, pp. 4340–4347.
[12] C. Wu, H. Mohsenian-Rad, and J. Huang, "Wind power integration via aggregator-consumer coordination: A game theoretic approach," in *Proceedings of IEEE PES Innovative Smart Grid Technologies*, Washington, DC, Jan. 2012, pp. 1–6.
[13] N. Li, L. Chen, and S. H. Low, "Optimal demand response based on utility maximization in power networks," in *Proceedings of IEEE Power Engineering Society General Meeting*, San Diego, CA, 2011, pp. 1–8.
[14] F. P. Kelly, "Charging and rate control for elastic traffic," *European Transactions on Telecommunications*, vol. 8, no. 1, pp. 33–37, 1997.
[15] Z. Fan, "Distributed charging of phevs in a smart grid," in *Proceedings of the IEEE International Conference on Smart Grid Communications*, Brussels.
[16] M. Vasirani and S. Ossowski, "A proportional share allocation mechanism for co-ordination of plug-in electric vehicle charging," *Engineering Applications of Artificial Intelligence*, vol. 26, no. 3, pp. 1185–1197, 2013.
[17] F. P. Kelly, A. K. Maulloo, and D. K. H. Tan, "Rate control in communication networks: Shadow prices, proportional prices and stability," *Journal of the Operational Research Society*, vol. 49, no. 3, pp. 237–252, 1998.
[18] H. Moulin, "The price of anarchy of serial, average and incremental cost sharing," *Economic Theory*, vol. 36, no. 3, pp. 379–405, 2008.
[19] T. Roughgarden, "Intrinsic robustness of the price of anarchy," *Communications of the ACM*, vol. 55, no. 7, pp. 116–123, 2012.
[20] ——, "Routing games," in *Algorithmic Game Theory*, N. Nisan, T. Roughgarden, E. Tardos, and V. V. Vazirani, Eds. Avenue of the Americas, NY: Cambridge University Press, 2007.
[21] R. Johari, "The price of anarchy and the design of scalable resource allocation mechanisms," in *Algorithmic Game Theory*, N. Nisan, T. Roughgarden, E. Tardos, and V. V. Vazirani, Eds. Avenue of the Americas, NY: Cambridge University Press, 2007.
[22] Q. Huang, M. Roozbelani, and M. Dahleh, "Efficiency-risk tradeoffs in dynamic oligopoly markets," in *51st IEEE Conference on Decision and Control*, Maui, HI, Dec. 2012, pp. 2388–2394.
[23] M. C. Caramanis and J. Foster, "Energy reserves and clearing in stochastic power markets:the case of plug-in-hybrid vehicle battery charging," in *49th IEEE Conference on Decision and Control*, Atlanta, GA, Dec. 2010, pp. 1037–1044.
[24] M. S. Bazaraa and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms*. NY: John Wiley and Sons, 1979.
[25] D. Bertsekas and J.N.Tsitsklis, *Parallel and Distributed Computation*. Englewood Cliffs, NJ: Prentice Hall, 1989.