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Impact of Irrational Consumers on Rational Consumers in a Smart Grid*

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Abstract— **Leveraging of flexibility in certain types of electric loads such as water heaters, washers, dryers, heating and air conditioning, and electric vehicles, offers an attractive and promising approach for large scale penetration of renewable generation and peak demand reduction. In price/market based approaches, consumers of flexible loads optimize their consumption schedules to meet their energy/power demands while minimizing their total costs. However, it may be that not all consumers will optimize their consumption schedules. We call such consumers "irrational consumers" and use the phrase "rational consumers" for those who optimize their consumption schedules. In this paper, we analyze the impact of irrational consumers on rational consumers in a stylized supply-demand balancing problem. We derive a condition which characterizes when the optimal utility derived by a rational consumer is lower than the case when all consumers act rationally. This condition uses parameters of the utility functions of consumers and the total consumption of the irrational consumers. The result offers some interesting insights into this phenomenon.**

I. INTRODUCTION

Large scale integration of renewable electricity production is a compelling technological objective to reduce carbon emissions arising from current fossil fuel based production infrastructure. Renewable sources of electric energy such as wind and solar are "variable", i. e., they are inherently uncertain, variable and non-dispatchable. This variability poses a significant challenge to power systems operations, especially at high penetration levels [1]. Power system capacity is largely dictated by the peak demand for power. This peak demand occurs only for a relatively small number of hours during any given year. A cursory examination of the load duration curve shows that a large fraction of the generation capacity goes unused most of the time as the peak demand occurs in a relatively small number of hours of the year [2].

Balancing of generation and consumption is a major operational constraint in power systems operations [3]. The traditional approach to this balancing requirement is by adjusting electricity production to meet (random, exogenous) demand. This adjustment is carried out via day-ahead and intra-day markets based scheduling of supply and additional real-time adjustments via frequency regulation. The remaining mismatch manifests itself via small frequency variations.

A paradigm shift is currently underway whereby demand adjustment also contributes to the supply-demand balancing. The idea behind demand adjustment is to leverage the potential flexibility in electric loads. This flexibility, naturally, varies from load to load and from consumer to consumer. Distributed control [4], [5], [6], [7], [8] offers a significant direction and opportunity for leveraging this load flexibility for reducing cost of renewable integration and need for capacity additions by reducing peak demand.

These adjustments in demand can be achieved via price signals sent to consumers [9], [4]. Design of such price signals is the subject of much current research under the smart grid theme. Regardless of the technique used, the ultimate result of any price based adjustment depends on consumer response to these signals. We use the term "rational consumers" to denote those that optimize their consumption schedules based on price considerations. However, not all consumers behave in this manner. In this paper, we take a simple model for such nonrational consumers. In particular, we use the term "irrational consumers" to denote those who make no alteration to their consumption in response to prices. Irrational behavior of consumers and its impact on the market is a research topic in the field of behavioral economics [10]. It is qualitatively shown that irrational behavior due to overconfidence affects market efficiency and investment decisions [11]. Again it is shown that irrational units would often be forced by a change in opportunities to respond rationally [12].

In the smart grid context, the importance of consumers' behavior has been studied in [13], [14], [15]. The year 2010 is highlighted as the year of the "Bakersfield Effect" [16] in smart grid circles. Certain patterns and trends of irrational behavior have been identified by behavioral economists. Using behavioral sciences and data analytics, utilities were able to motivate changes in customer energy usage behavior and reduce utility bills [17]. In [18], two different incentive schemes on demand-side management (DSM) techniques have been designed; one to encourage and another to discourage adoption of DSM. Leveraging results from evolutionary game theory, it has been shown that in the first case, a large amount of agents adopt DSM whereas in the second case, only a fraction of the agents' population uses DSM and the system reaches mixed equilibrium. In [19], optimal power consumption policies in a system having two types of loads (flexible and fixed) have been computed for co-operative and non-cooperative market structures under real time pricing. Then efficiency-risk trade off has been studied in both the cases using numerical simulations.

In this paper, we analyze the impact of irrational consumers on rational consumers. In particular, we pose the

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question: under what conditions would a rational consumer receive a lower total utility due to the presence of irrational consumers as compared to the ideal case where all consumers are rational. We analyze this question in a stylized problem formulation. Here we take the case of distributed control with price taking flexible load consumers where price is determined by proportional allocation mechanism [9]. We obtain a necessary and sufficient condition to characterize this phenomenon. This condition uses parameters of the utility functions of consumers and the total consumption of the irrational consumers.

II. Problem Formulation

We begin by briefly presenting our problem formulation which has been described in detail in [9]. We consider a residential area with thermal and renewable generators, uncontrolled loads, and flexible loads. Uncontrolled loads are those loads whose consumption schedules can not be varied or controlled. Flexible loads are those loads whose consumption schedules can be shifted to different times or whose consumption levels at a particular time can be varied.

We start with the notations:

- The time horizon is divided into T discrete slots and indexed by $t \in \mathbb{T} := (1,2,...,T)$. (Typical value of T 6-8 hours.)
- Flexible load consumers are denoted by $\mathbb{N} := (1,2,...,N)$ and indexed by i. For simplicity, we assume that there are N flexible load consumers each having one flexible load. Each flexible load consumer possesses a smart meter with two way communication capability.
- $d_i(t)$: the power consumption of the i-th flexible load at time t.
- $c(t)$: the total scheduled power generation of all the thermal power plants at time t.
- $w(t)$: the total predicted power supply of the renewable generators at time t.
- $n(t)$: total power consumption of all uncontrolled loads at time t.

We assume there is a traditional day ahead market, where the supply is scheduled based on demand predictions. During the operating day, the control authority obtains a better forecast of renewable generation and accordingly, it sets an electric energy price for time period T in order to balance supply with demand, i.e.,

$$c(t) + w(t) = n(t) + \sum_{i=1}^{N} d_i(t) \quad \forall t \in \mathbb{T}. \tag{1}$$

We define

$$v(t) := c(t) + w(t) - n(t). \tag{2}$$

So, (1) becomes

$$v(t) = \sum_{i=1}^{N} d_i(t) \quad \forall t \in \mathbb{T}. \tag{3}$$

Now we assume that

$$v(t) > 0 \tag{4}$$

for all t. This means that the scheduled thermal generation and renewable production are sufficient to meet the uncontrollable demand. The supply demand balancing is done by adjusting the power consumption of the flexible loads. We do not allow load to grid power transfer, i.e.,

$$d_i(t) \geq 0 \tag{5}$$

for all i and t.

Let U_i denote the utility function of a flexible load i in monetary units. The utility function U_i is assumed to be a concave, continuously differentiable and strictly increasing function. The flexible loads satisfy the following constraints:

- The power consumption is bounded above and below by d_i^{max} and d_i^{min} respectively:
$$d_i^{min} \leq d_i(t) \leq d_i^{max} \tag{6}$$
where d_i^{min} is nonnegative as per (5).
- The total energy consumption of each flexible load is bounded below and above by q_i^{min} and q_i^{max}:
$$q_i^{min} \leq \sum_{t=1}^{T} d_i(t) \leq q_i^{max}. \tag{7}$$

As the above constraints are linear, we can write them together in the following form

$$\sum_{t=1}^{T} \gamma_i^m(t) d_i(t) \leq b^m \quad \forall m \in \mathbb{M},\ i \in \mathbb{N} \tag{8}$$

where the set $\mathbb{M} := (1,2,...,M)$ is the set of constraints indexed by m.

III. Centralized Control

At first we consider the case where a central control authority intends to maximize the total utility of consumers while satisfying power balance constraint at each time. Thus, the control authority's objective is to

$$\underset{d_i(t)}{maximize} \quad V = \sum_{i=1}^{N} \sum_{t=1}^{T} U_i(d_i(t)) \tag{9}$$

subject to (3) and (8).

We assume that the convex set produced by the set of constraints defined by (3) and (8) is nonempty. Since this is a concave optimization problem with convex inequality constraint functions, the above assumption will ensure that global maxima exist for this problem [20]. The central control authority can compute the optimal solution using the well-known KKT conditions [20]. See [9] for more details.

IV. Distributed Control with Rational Flexible Load Consumers

The centralized control approach faces the following drawbacks:

1) The central authority should know utility functions and operational constraints of all the flexible load consumers at all times to solve the problem. The consumers may not want to disclose these information to the authority.

2) The consumers may want to handle the power consumption of their loads independently.
3) Considering large numbers of flexible loads, the dimension of the optimization problem may become very large.

Therefore, the distributed control technique described below is more attractive [9].

Let $k_i(t)$ be the monetary value associated with consumption $d_i(t)$. Here at first the control authority, after getting the values of $c(t)$, $w(t)$ and $n(t)$, calculates $v(t)$ according to (2) for all t and broadcasts the value to all the consumers. Each consumer then sends its $k_i(t)$ to the central control authority for all t. The central control authority calculates $\sum_{i=1}^{N} k_i(t)$ for each time t and computes price according to the following formula

$$p(t) = \frac{\sum_{i=1}^{N} k_i(t)}{v(t)} \tag{10}$$

where $v(t) \neq 0$ as per (4). The allocation of $d_i(t)$ to the i-th consumer is

$$d_i(t) = \frac{k_i(t)}{p(t)} \tag{11}$$

for all i and t. This mechanism of demand allocation is called "proportional allocation mechanism" [21]. We assume all the consumers to be rational who will maximize their utility functions and minimize the costs. More specifically, each consumer i will

$$\underset{k_i(t)}{maximize} \quad L_i = \sum_{t=1}^{T} U_i(\frac{k_i(t)}{p(t)}) - \sum_{t=1}^{T} k_i(t) \tag{12}$$

subject to the constraints

$$\sum_{t=1}^{T} \gamma_i^m(t) \frac{k_i(t)}{p(t)} \leq b^m \quad \forall m \in \mathbb{M}. \tag{13}$$

We define i-th consumer's strategy as $k_i = (k_i(t))$ and price in the system as $p = (p(t))$ for all t. We say that (k_i^*, p^*) is a *competitive equilibrium* if

$$L_i(k_i^*, p^*) \geq L_i(\hat{k}_i, \hat{p}) \quad \forall\, i \in \mathbb{N} \tag{14}$$

where $\hat{k}_i$ and $\hat{p}$ are all possible consumption schedules. We assume that the convex set produced by the set of constraints defined by (13) is non-empty. As the objective function is concave and inequality constraint functions are convex, global maxima for the l-th consumer exist. Now it has been shown that the distributed problem has a competitive equilibrium [9]. The KKT conditions of this distributed problem are also identical to the KKT conditions of the centralized control problem defined in section III with Lagrange multiplier corresponding to the equality constraint equal to $p(t)$. So the solution of distributed control problem will maximize the aggregate centralized utility [9]. Also a distributed algorithm is developed to jointly compute the price and consumption schedule [9].

V. Distributed Control with Some Irrational Flexible Load Consumers

Let us assume that there are some consumers who do not change their consumption plans according to the changes in price signal. They are labeled as irrational consumers. The irrational consumers are indexed by $j \in \mathbb{J}$ where set $\mathbb{J}$ contains J members . The rational consumers are indexed by $l \in \mathbb{L}$ where set $\mathbb{L}$ contains L members. So according to our definition, $J + L = N$. The monetary value associated with demand $\tilde{d}_l(t)$ of a rational consumer is $\tilde{k}_l(t)$ and that with demand $\tilde{d}_j(t)$ of a irrational consumer is $\tilde{k}_j(t)$ (which will remain unchanged with respect to the change in the price signal). So the formula for price with the new notations is as follows:

$$p(t) = \frac{\sum_{l \in \mathbb{L}} \tilde{k}_l(t) + \sum_{j \in \mathbb{J}} \tilde{k}_j(t)}{v(t)} \tag{15}$$

where allocation of demands are

$$\tilde{d}_l(t) = \frac{\tilde{k}_l(t)}{p(t)} \tag{16}$$

$$\tilde{d}_j(t) = \frac{\tilde{k}_j(t)}{p(t)}. \tag{17}$$

Here only the L rational consumers will try to maximize their objectives subject to the operational constraints of their loads. So each of them will

$$\underset{\tilde{k}_l(t)}{maximize} \quad L_l = \sum_{t=1}^{T} U_l(\frac{\tilde{k}_l(t)}{p(t)}) - \sum_{t=1}^{T} \tilde{k}_l(t) \tag{18}$$

subject to the constraints

$$\sum_{t=1}^{T} \gamma_l^m(t) \frac{\tilde{k}_l(t)}{p(t)} \leq b^m \quad \forall m \in \mathbb{M}. \tag{19}$$

We assume that the convex set produced by the set of constraints defined by (19) is non-empty. Now as the objective function is concave and inequality constraint functions are convex, global maxima for the l-th consumer exist. Maxima can be found from the KKT conditions. In order to find the KKT conditions, let us define

$$\hat{L}_l = \sum_{t=1}^{T} U_l(\frac{\tilde{k}_l(t)}{p(t)}) - \sum_{t=1}^{T} \tilde{k}_l(t) - \sum_{m=1}^{M} \mu_m^l (\sum_{t=1}^{T} \gamma_l^m(t) \frac{\tilde{k}_l(t)}{p(t)} - b^m) \tag{20}$$

where μ_m^l is the Lagrange multiplier. Now taking partial derivatives with respect to $\tilde{k}_l(t)$ and writing the complementary slackness condition for the inequality constraints, we get the following KKT conditions:

$$U_l'(\frac{\tilde{k}_l(t)}{p(t)}) - p(t) - \sum_{m=1}^{M} \mu_m^l \gamma_l^m(t) = 0 \tag{21}$$

$$\mu_m^l (\gamma_l^m(t) \frac{\tilde{k}_l(t)}{p(t)} - b^m) = 0 \tag{22}$$

$$\mu_m^l \geq 0 \tag{23}$$

So, this implies that distributed control will achieve supply-demand balancing even if some consumers act irrationally provided the set produced by all the constraints are non-empty.

Now let us consider the following hypothetical centralized optimization problem:

$$\underset{\tilde{d}_l(t)}{maximize} \quad Q = \sum_{l\in\mathbb{L}}\sum_{t=1}^{T} U_l(\tilde{d}_l(t)) \tag{24}$$

subject to the operational constraints

$$\sum_{t=1}^{T} \gamma_l^m(t)\tilde{d}_l(t) \le b^m \quad \forall m \in \mathbb{M},\ l \in \mathbb{L} \tag{25}$$

and the supply-demand balance equation

$$v(t) = \sum_{l\in\mathbb{L}} \tilde{d}_l(t) + \sum_{j\in\mathbb{J}} \tilde{d}_j(t) \quad \forall t \in \mathbb{T}. \tag{26}$$

This is a concave optimization problem with convex inequality constraint functions. We assume that the convex set produced by the set of constraints defined by (26) is non-empty. Then global maxima exists and can be found using KKT conditions. In order to calculate the KKT conditions, let us define

$$\hat{Q} = \sum_{l\in\mathbb{L}}\sum_{t=1}^{T} U_l(\tilde{d}_l(t)) - \sum_{t=1}^{T}\lambda(t)(v(t) - \sum_{l\in\mathbb{L}}\tilde{d}_l(t) - \sum_{j\in\mathbb{J}}\tilde{d}_j(t)) - \sum_{l\in\mathbb{L}}\sum_{m=1}^{M}\underline{\mu}_m^l(\sum_{t=1}^{T}\gamma_l^m(t)\tilde{d}_l(t) - b^m) \tag{27}$$

where $\lambda(t)$ and $\underline{\mu}_m^l$ are the Lagrange multipliers. Now taking partial derivatives with respect to $\tilde{d}_l(t)$ and $\lambda(t)$ and writing the complementary slackness condition for the inequality constraints, we get the following KKT conditions:

$$U_l'(\tilde{d}_l(t)) - \lambda(t) - \sum_{m=1}^{M}\underline{\mu}_m^l\gamma_l^m(t) = 0 \tag{28}$$

$$v(t) = \sum_{l\in\mathbb{L}}\tilde{d}_l(t) + \sum_{j\in\mathbb{J}}\tilde{d}_j(t) \tag{29}$$

$$\underline{\mu}_m^l(\sum_{t=1}^{T}\gamma_l^m(t)\tilde{d}_l(t) - b^m) = 0 \tag{30}$$

$$\underline{\mu}_m^l \ge 0 \tag{31}$$

for all $l \in \mathbb{L}, t \in \mathbb{T}, m \in \mathbb{M}$. Now if we compare equations (21)-(23) with that of (28)-(31), we see that the distributed control problem will have same global maxima as that of the hypothetical centralized problem having $\lambda(t) = p(t)$. In that case, $\mu_m^l = \underline{\mu}_m^l$.

A. *Algorithm to Compute the Price and Consumption Schedule*

Next we develop a distributed algorithm, where the central control authority and flexible load consumers jointly compute their power consumption schedule and the price .

1) The control authority computes the value of $v(t)$ as per (2) for all t and broadcasts to all the flexible load consumers.
2) Rational and irrational consumers send their $\tilde{k}_l(t)$ and $\tilde{k}_j(t)$ for all t to the control authority.
3) The authority adds all the $\tilde{k}_l(t)$s and $\tilde{k}_j(t)$s to obtain $\sum_{l\in\mathbb{L}}\tilde{k}_l(t)$ and $\sum_{j\in\mathbb{J}}\tilde{k}_j(t)$. It then computes $p(t)$ according to (15) and broadcasts to all the consumers.
4) The rational consumers update their consumption schedule according to

$$\bar{\tilde{k}}_l^{(f+1)}(t) = \tilde{k}_l^f(t) + \gamma(U_l'(\frac{\tilde{k}_l(t)}{p(t)}) - p(t)) \tag{32}$$

$$\tilde{k}_l^{(f+1)}(t) = [\bar{\tilde{k}}_l^{(f+1)}(t)]^{S_l} \tag{33}$$

where $\gamma > 0$ is a constant step size, f is the number of iterations and $[.]^{S_l}$ is projection onto the set S_l produced by constraints (26). The smart meters associated with irrational consumers send the same value without following the price signal.

5) The process from step (3)-step (4) is continued until convergence is achieved. When γ is small enough, the algorithm converges [22].

VI. Impact of Irrational Consumers on the Utility Functions of the Remaining Rational Consumers

As the solution to the distributed control problem with some irrational consumers does not maximize the centralized aggregate utility of section III, there will be overall utility loss due to irrational behavior. Here we focus our attention on the impact on the utility derived by the rational consumers due to the fact that some of the consumers are irrational. Specifically, we are interested in the question: Will the total utility derived by a rational consumer be reduced (or increased) due to the presence of some irrational consumers as compared to the case where all consumers act rationally to optimize their consumption schedule?

Our goal is to obtain analytically tractable results in order to gain qualitative insights into this question. To facilitate this, we will consider those flexible loads (e. g., electric vehicles, washers, dryers, etc.) for which the utility function is inter-temporal and the total utility over time T is a function of the total energy consumed, i.e., $\sum_{t=1}^{T} U_i(d_i(t)) = U_i(\sum_{t=1}^{T} d_i(t))$.

We make two assumptions:

A1: The utility functions of the flexible load consumers are quadratic (with negative coefficient of the quadratic term to maintain concavity). (For example, the utility of an electric vehicle= $\sum_{t=1}^{T} U_i(d_i(t)) = U_i(\sum_{t=1}^{T} d_i(t)) = K_i - (q_i^{max} - \sum_{t=1}^{T} d_i(t))^2$ So the utility increases quadratically from $K_i - (q_i^{max})^2$ to K_i with increase in energy consumption.)

A2: The inequality constraints defined by equations (8), (26), (13), and (19) are non-binding and the optimal solution is in the interior of the set produced by the constraint equations.

The main result of the paper is stated next.

Theorem 6.1: Define the following scenarios:

Scenario I: All consumers are rational; the optimal consumption schedule is given by solutions of (12) and (13) in section IV and

Scenario II: Some of the consumers are irrational; the optimal consumption schedule is given by solutions of (18) and (19) in section V.

Suppose assumptions **A1** and **A2** are satisfied. Suppose the coefficients of the quadratic utility functions of rational consumers are denoted by $-a_l$, b_l, and c_l and those of irrational consumers are denoted by $-a_j$, b_j, and c_j.

Then the optimal total utility of a rational consumer l in Scenario I and Scenario II satisfies the following property:

$$\sum_{t=1}^{T} U_l(d_l(t)) \geq \sum_{t=1}^{T} U_l(\tilde{d}_l(t)) \quad (34)$$

iff

$$\sum_{j\in\mathbb{J}}\sum_{t=1}^{T} \tilde{d}_j(t) \geq \frac{\sum_{t=1}^{T} v(t)\sum_{j\in\mathbb{J}}\frac{1}{2a_j} - \sum_{l\in\mathbb{L}}\frac{b_l}{2a_l}\sum_{j\in\mathbb{J}}\frac{1}{2a_j} + \sum_{j\in\mathbb{J}}\frac{b_j}{2a_j}\sum_{l\in\mathbb{L}}\frac{1}{2a_l}}{\sum_{l\in\mathbb{L}}\frac{1}{2a_l} + \sum_{j\in\mathbb{J}}\frac{1}{2a_j}}. \quad (35)$$

Further, if we assume that the utility functions are the same for all consumers i.e. $a_l = a_j$, $b_l = b_j$, and $c_l = c_j$ for all l and j ,

$$\sum_{t=1}^{T} U(d_l(t)) \geq \sum_{t=1}^{T} U(\tilde{d}_l(t)) \quad (36)$$

$\Leftrightarrow$

$$\frac{\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}\tilde{d}_j(t)}{J} \geq \frac{\sum_{t=1}^{T} v(t)}{N}. \quad (37)$$

Proof: As we have assumed quadratic utility, the distributed control problems of both scenario I and II have same solution as that of two centralized problems described earlier. We use those associated centralized control problems to prove the result.

A. Scenario I: all the flexible load consumers are rational

Here,

$$\sum_{t=1}^{T} U_i(d_i(t)) = U_i(\sum_{t=1}^{T} d_i(t))$$
$$= -a_i(\sum_{t=1}^{T} d_i(t))^2 + b_i(\sum_{t=1}^{T} d_i(t)) + c_i. \quad (38)$$

So, the centralized problem is to

$$\underset{d_i(t)}{maximize} \quad V = \sum_{i=1}^{N}(-a_i(\sum_{t=1}^{T} d_i(t))^2 + b_i(\sum_{t=1}^{T} d_i(t)) + c_i) \quad (39)$$

subject to the supply-demand balance equation

$$v(t) = \sum_{i=1}^{N} d_i(t) \quad \forall t \in \mathbb{T}. \quad (40)$$

In order to compute the KKT conditions, let us define

$$\hat{V} = \sum_{i=1}^{N}(-a_i(\sum_{t=1}^{T} d_i(t))^2 + b_i(\sum_{t=1}^{T} d_i(t)) + c_i + \sum_{t=1}^{T}\lambda(t)(v(t) - \sum_{i=1}^{N} d_i(t)). \quad (41)$$

Taking partial derivative with respect to $d_i(t)$

$$-2a_i(\sum_{t=1}^{T} d_i(t)) + b_i = \lambda(t). \quad (42)$$

Here, $\lambda(t) = \lambda$ (independent of time). Therefore,

$$\sum_{t=1}^{T} d_i(t) = \frac{\lambda - b_i}{-2a_i}. \quad (43)$$

So, taking summation over all the consumers' consumption

$$\sum_{i=1}^{N}\sum_{t=1}^{T} d_i(t) = -\lambda\sum_{i=1}^{N}\frac{1}{2a_i} + \sum_{i=1}^{N}\frac{b_i}{2a_i}. \quad (44)$$

Again from (40) we can write,

$$\sum_{i=1}^{N}\sum_{t=1}^{T} d_i(t) = \sum_{t=1}^{T} v(t). \quad (45)$$

Combining the above two equations, we can find the value of λ as

$$\lambda = \frac{\sum_{t=1}^{T} v(t) - \sum_{i=1}^{N}\frac{b_i}{2a_i}}{\sum_{i=1}^{N}\frac{1}{-2a_i}}. \quad (46)$$

Putting the value of λ in (43), we get the total energy consumption of the i-th consumer as

$$\sum_{t=1}^{T} d_i(t) = \frac{\frac{\sum_{t=1}^{T} v(t) - \sum_{i=1}^{N}\frac{b_i}{2a_i}}{\sum_{i=1}^{N}\frac{1}{-2a_i}} - b_i}{-2a_i}. \quad (47)$$

Next, we consider the Scenario II.

B. Scenario II: Some of the consumers are irrational

The hypothetical equivalent centralized optimization problem is to

$$\underset{\tilde{d}_l(t)}{maximize} \quad Q = \sum_{l\in\mathbb{L}}(-a_l(\sum_{t=1}^{T}\tilde{d}_l(t))^2 + b_l(\sum_{t=1}^{T}\tilde{d}_l(t)) + c_l) \quad (48)$$

subject to the supply-demand balance equation

$$v(t) = \sum_{l\in\mathbb{L}}\tilde{d}_l(t) + \sum_{j\in\mathbb{J}}\tilde{d}_j(t) \quad \forall t \in \mathbb{T}. \quad (49)$$

To compute the KKT conditions, let us define

$$\hat{Q} = \sum_{l\in\mathbb{L}}(-a_l(\sum_{t=1}^{T}\tilde{d}_l(t))^2 + b_l(\sum_{t=1}^{T}\tilde{d}_l(t)) + c_l + \sum_{t=1}^{T}\lambda(t)(v(t) - \sum_{l\in\mathbb{L}}\tilde{d}_l(t) - \sum_{j\in\mathbb{J}}\tilde{d}_j(t)). \quad (50)$$

Taking partial derivative with respect to $\tilde{d}_l(t)$

$$-2a_l(\sum_{t=1}^{T}\tilde{d}_l(t)) + b_l = \lambda(t). \quad (51)$$

Here also the $\lambda(t)$ is independent of time and is defined as λ. Therefore,

$$\sum_{t=1}^{T}\tilde{d}_l(t) = \frac{\lambda - b_l}{-2a_l}. \quad (52)$$

So taking summation over all the rational consumers' consumption

$$\sum_{l\in\mathbb{L}}\sum_{t=1}^{T}\tilde{d}_l(t)=-\lambda\sum_{l\in\mathbb{L}}\frac{1}{2a_l}+\sum_{l\in\mathbb{L}}\frac{b_l}{2a_l}. \tag{53}$$

Again from (49) we can write,

$$\sum_{l\in\mathbb{L}}\sum_{t=1}^{T}\tilde{d}_l(t)=\sum_{t=1}^{T}v(t)-\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}\tilde{d}_j(t). \tag{54}$$

So combining the above two equations, we get

$$\lambda=\frac{\sum_{t=1}^{T}v(t)-\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}\tilde{d}_j(t)-\sum_{l\in\mathbb{L}}\frac{b_l}{2a_l}}{\sum_{l\in\mathbb{L}}\frac{1}{-2a_l}}. \tag{55}$$

Putting the value of λ in (49), we get the energy consumption of the l-th rational consumer as

$$\sum_{t=1}^{T}\tilde{d}_l(t)=\frac{\frac{\sum_{t=1}^{T}v(t)-\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}d_j(t)-\sum_{l\in\mathbb{L}}\frac{b_l}{2a_l}}{\sum_{l\in\mathbb{L}}\frac{1}{-2a_l}}-b_l}{-2a_l}. \tag{56}$$

C. Comparison of utility between Scenario I and II

Let us compare the utility of a rational consumer l between two cases. Now from (47), the energy consumption of the consumer l across time T in case I is

$$\sum_{t=1}^{T}d_l(t)=\frac{\frac{\sum_{t=1}^{T}v(t)-\sum_{i=1}^{N}\frac{b_i}{2a_i}}{\sum_{i=1}^{N}\frac{1}{-2a_i}}-b_l}{-2a_l}. \tag{57}$$

In order to compare $\sum_{t=1}^{T}\tilde{d}_l(t)$ in (56) with $\sum_{t=1}^{T}d_l(t)$ in (57), let us define

$$x=\sum_{t=1}^{T}v(t)-\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}\tilde{d}_j(t)-\sum_{l\in\mathbb{L}}\frac{b_l}{2a_l} \tag{58}$$

and

$$y=\sum_{l\in\mathbb{L}}\frac{1}{2a_l}. \tag{59}$$

So,

$$\sum_{t=1}^{T}d_l(t)-\sum_{t=1}^{T}\tilde{d}_l(t)$$

$$=\frac{1}{2a_l}\left(\frac{x+\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}d_j(t)-\sum_{j\in\mathbb{J}}\frac{b_j}{2a_j}}{y+\sum_{j\in\mathbb{J}}\frac{1}{2a_j}}-\frac{x}{y}\right) \tag{60}$$

$$=\frac{1}{2a_l}\left(\frac{y(\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}d_j(t)-\sum_{j\in\mathbb{J}}\frac{b_j}{2a_j})-x(\sum_{j\in\mathbb{J}}\frac{1}{2a_j})}{(y+\sum_{j\in\mathbb{J}}\frac{1}{2a_j})y}\right) \tag{61}$$

Therefore,

$$\sum_{t=1}^{T}d_l(t)\geq\sum_{t=1}^{T}\tilde{d}_l(t) \tag{62}$$

$\Leftrightarrow$

$$y\left(\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}\tilde{d}_j(t)-\sum_{j\in\mathbb{J}}\frac{b_j}{2a_j}\right)\geq x\left(\sum_{j\in\mathbb{J}}\frac{1}{2a_j}\right). \tag{63}$$

Substituting the values of x and y in the above expression and upon algebraic simplification, we get

$$\sum_{t=1}^{T}d_l(t)\geq\sum_{t=1}^{T}\tilde{d}_l(t) \tag{64}$$

$\Leftrightarrow$

$$\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}\tilde{d}_j(t)\geq$$

$$\frac{\sum_{t=1}^{T}v(t)\sum_{j\in\mathbb{J}}\frac{1}{2a_j}-\sum_{l\in\mathbb{L}}\frac{b_l}{2a_l}\sum_{j\in\mathbb{J}}\frac{1}{2a_j}+\sum_{j\in\mathbb{J}}\frac{b_j}{2a_j}\sum_{l\in\mathbb{L}}\frac{1}{2a_l}}{\sum_{l\in\mathbb{L}}\frac{1}{2a_l}+\sum_{j\in\mathbb{J}}\frac{1}{2a_j}}. \tag{65}$$

Again as the utility functions are continuously differentiable and strictly increasing,

$$\sum_{t=1}^{T}d_l(t)\geq\sum_{t=1}^{T}\tilde{d}_l(t) \tag{66}$$

$\Leftrightarrow$

$$\sum_{t=1}^{T}U_l(d_l(t))\geq\sum_{t=1}^{T}U_l(\tilde{d}_l(t)). \tag{67}$$

Therefore,

$$\sum_{t=1}^{T}U_l(d_l(t))\geq\sum_{t=1}^{T}U_l(\tilde{d}_l(t)) \tag{68}$$

$\Leftrightarrow$

$$\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}\tilde{d}_j(t)\geq$$

$$\frac{\sum_{t=1}^{T}v(t)\sum_{j\in\mathbb{J}}\frac{1}{2a_j}-\sum_{l\in\mathbb{L}}\frac{b_l}{2a_l}\sum_{j\in\mathbb{J}}\frac{1}{2a_j}+\sum_{j\in\mathbb{J}}\frac{b_j}{2a_j}\sum_{l\in\mathbb{L}}\frac{1}{2a_l}}{\sum_{l\in\mathbb{L}}\frac{1}{2a_l}+\sum_{j\in\mathbb{J}}\frac{1}{2a_j}}. \tag{69}$$

Let us assume that all the consumers have same utility functions. Then the above condition simplifies to

$$\sum_{t=1}^{T}U_l(d_l(t))\geq\sum_{t=1}^{T}U_l(\tilde{d}_l(t)) \tag{70}$$

$\Leftrightarrow$

$$\frac{\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}\tilde{d}_j(t)}{J}\geq\frac{\sum_{t=1}^{T}v(t)}{N}. \tag{71}$$

■

We can now offer some qualitative interpretations of the above result.

- First, the necessary and sufficient condition is the same regardless of the specific rational consumer under consideration. This is an interesting property of the result which is not intuitively obvious and may be related to the specific allocation mechanism. Thus, either all rational consumers increase their utility or decrease their utility but there cannot be a mix.
- The case of identical utility function parameters is easier to understand. The left hand side of the inequality (71) is the average energy consumption schedule of all the irrational consumers over the time horizon $[0,T]$. As $v(t)$ has to be balanced by power consumption of N number of flexible load consumers, the right hand side of the above expression can be interpreted as the average

of flexible consumers' balancing energy requirement over the same time horizon. Thus, the main result shows that the rational consumers will receive reduced utility if the irrational consumers consume more than the average consumption in the all rational consumer case, i. e., more than their fair share.

- Consider the case of J irrational consumers such that

$$\frac{\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}\tilde{d}_j(t)}{J} \leq \frac{\sum_{t=1}^{T}v(t)}{N}. \tag{72}$$

According to the main result, the utilities of each of the remaining rational consumers will increase. Now suppose we have one more irrational consumer, i. e., there are $(J+1)$ irrational consumers . Now let the new set of irrational consumers is defined as $\hat{\mathbb{J}}$ and $(J+1)$-th consumers' power consumption schedule is such that,

$$\sum_{t=1}^{T}\tilde{d}_{(J+1)}(t) \geq (J+1)\frac{\sum_{t=1}^{T}v(t)}{N} - \frac{\sum_{j\in\mathbb{J}}\sum_{t=1}^{T}\tilde{d}_j(t)}{J} \tag{73}$$

Then

$$\frac{\sum_{j\in\hat{\mathbb{J}}}\sum_{t=1}^{T}\tilde{d}_j(t)}{(J+1)} \geq \frac{\sum_{t=1}^{T}v(t)}{N} \tag{74}$$

Therefore, the utilities of the rational consumers will decrease. Since the $(J+1)$-th consumer's initial consumption schedule helps balancing the system, all of the rational consumers will have to reduce their consumption schedules reducing their utilities.

VII. Conclusion and Future Work

In this paper, we have investigated supply-demand balancing for renewable integration by distributed control by leveraging flexible loads. We explored distributed solution with some irrational flexible load consumers and analyzed the impact of irrational behavior on the remaining rational consumers. We are currently exploring several questions in this general direction:

1) How can rational behavior among consumers be encouraged by suitably designing price/reward mechanism? What will motivate consumers to reveal their preferences?
2) How does the analysis change when some of the consumers are price anticipating and some are irrational?
3) How can we detect irrational behavior?
4) Is there a suitable pricing scheme that treats irrational consumers and rational consumers in a fair manner?

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