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Sharing Storage in a Smart Grid: A Coalitional Game Approach

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***Abstract*—Sharing economy is a transformative socio-economic phenomenon built around the idea of sharing underused resources and services, *e.g.* transportation and housing, thereby reducing costs and extracting value. Anticipating continued reduction in the cost of electricity storage, we look into the potential opportunity in electrical power system where consumers share storage with each other. We consider two different scenarios. In the first scenario, consumers are assumed to already have individual storage devices and they explore cooperation to minimize the realized electricity consumption cost. In the second scenario, a group of consumers is interested to invest in joint storage capacity and operate it cooperatively. The resulting system problems are modeled using cooperative game theory. In both cases, the cooperative games are shown to have non-empty cores and we develop efficient cost allocations in the core with analytical expressions. Thus, sharing of storage in cooperative manner is shown to be very effective for the electric power system.**

***Index Terms*—Storage Sharing, Cooperative Game Theory, Cost Allocation**

I. INTRODUCTION

A. Motivation

The sharing economy is disruptive and transformative socio-economic trend that has already impacted transportation and housing [1]. People rent out (rooms in) their houses and use their cars to provide transportation services. The business model of sharing economy leverages under utilized resources. Like these sectors, many of the resources in electricity grid is also under-utilized or under-exploited. There is potential benefit in sharing the excess generation by rooftop solar panels, sharing flexible demand, sharing unused capacity in the storage services, etc. Motivated by the recent studies [2] predicting a fast drop in battery storage prices, we focus on sharing electric energy storage among consumers.

B. Literature Review

Storage prices are projected to decrease by more than 30% by 2020 [2]. The arbitrage value and welfare effects of storage in electricity markets has been explored in literature. In [3], the value of storage arbitrage was studied in deregulated markets. In [4], the authors studied the role of storage in wholesale electricity markets. The economic viability of the storage elements through price arbitrage was examined in [5]. Agent-based models to explore the tariff arbitrage opportunities for residential storage systems were introduced in [6]. In [7], [8], authors address the optimal control and coordination of energy storage. In [9], the consumers manage and schedule their demands controlling their storage systems. All these works explore the economic value of storage to an individual, not for shared services.

Game theory is used to model interaction of multi-agent systems. In the field of smart power grid and renewable integration, there are many papers that use different types of game theory models, see for example, [10], [11], [12], [13]. Relevant to this paper, sharing of storage among firms has been analyzed using non-cooperative game theory in [14], where the interaction among the consumers is mediated by designing a spot market and, as a result, the consumers engage in strategic behavior. In addition, an aggregator manages the coordination among the strategic consumers and under a certain alignment condition of consumptions, the agents reach a solution. Here in our paper, we explore sharing storage in a cooperative manner among consumers. We model the interaction among the consumers using cooperative game theory. Our model does not require either to implement a spot market or to satisfy any alignment condition. Moreover, we consider two different sharing scenarios and develop stabilizing cost allocations for the joint electricity consumption costs in both the scenarios. Cooperative game theory has significant potential to model resource sharing effectively [15]. Cooperation and aggregation of renewable energy sources bidding in a two settlement market to maximize expected and realized profit has been analyzed using cooperative game theory in [16]–[18]. Under a cooperative set-up, the cost allocation to all the agents is a crucial task. A framework for allocating cost in a fair and stable way was introduced in [19]. In [20], the authors use novel cooperative game theoretic operational strategies among the micro-grids of a distribution network. Cooperative game theoretic analysis of multiple demand response aggregators in a virtual power plant and their cost allocation has been tackled in [21]. In [22], sharing opportunities of photovoltaic systems (PV) under various billing mechanisms were explored using cooperative game theory.

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C. Contributions and Paper Organization

In this paper, we investigate the sharing of storage systems in a time of use (TOU) price set-up using cooperative game theory. We consider two scenarios. In the first one, a group of consumers already own storage systems and they are willing to operate all together to minimize their electricity consumption cost. In a second scenario, a group of consumers wish to invest in a shared common storage system and get benefit for long term operation in a cooperative manner. We model both the cases using cooperative game theory. We prove that the resulting games developed have non-empty cores –*i.e.* cooperation is shown to be beneficial in both the cases. We also derive closed-form and easy to compute expressions for cost allocations in the core in both the cases. Our results suggest that sharing of electricity storage in a cooperative manner is an effective way to amortize storage costs and to increase its utilization. In addition, it can be very much helpful for consumers and at the same time to integrate renewables in the system, because off-peak periods correspond to large presence of renewables that can be stored for consumption during peak periods.

It is also important to note that the rapid response of energy storage systems is an important feature for ensuring stability of the grid when unexpected changes in generation or demand occur. Storage systems are potentially powerful resources to improve the dynamic behavior of power systems. But in this work, we concentrate on the economic benefit of storage sharing and do not analyze its potential for contributing to the dynamic behavior of the system.

The remainder of the paper is organized as follows. In Section II, we formulate the cooperative storage problems. A brief review of cooperative game theory is presented in Section III. In Section IV, we state and explain our main results. A case study illustrating our results using real data from Pecan St. Project is presented in Section VII. Finally, we conclude the paper in Section VIII.

II. Problem Formulation

A. System Model

We consider a set of consumers indexed by $i \in \mathcal{N} := \{1, 2, \ldots, N\}$. The consumers invest in storage. The consumers cooperate and share their storage with each other. We consider two scenarios here. In Scenario I, the consumers already have purchased storage and are assumed to operate with suitable electric grid connections of all the storage devices. In Scenario II, the consumers invest in a common storage device and their objective is long-term benefit. The houses or firms of the collaborating consumers need to be physically connected to the common storage device and there should be a single smart energy meter for the group. In both cases, we assume that the cost of the necessary electrical connection between the consumers for effective sharing is included as part of the cost of acquiring the storage device, and the physical infrastructure provides the required capacity. We ignore in the model the capacity constraints, topology or losses in the connecting network. The configuration of the scenarios with three consumers are depicted in Figure 1.

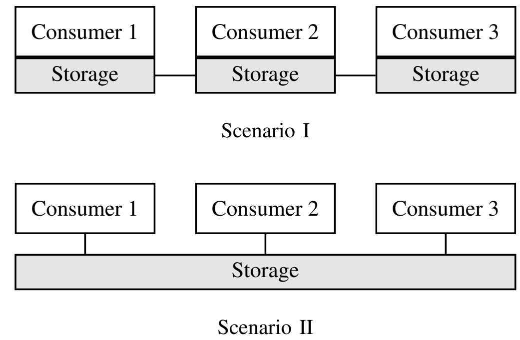

Fig. 1. Configuration of three consumers in the two analyzed scenarios

Examples of the situations considered here include consumers in an industrial park, office buildings on a campus, or homes in a residential complex. It is interesting to note that some battery manufacturers, e.g., Sonnen in Germany, that are already promoting the concept of sharing storage among neighboring households as a business model [23].

B. Cost of Energy Consumption with Storage

Each day is divided into two periods –peak and off-peak. There is a time-of-use pricing. The peak and off-peak period prices are denoted by π_h and π_ℓ respectively. The prices are fixed and known to all the consumers.

Let π_i be the daily capital cost of storage of the consumer $i \in \mathcal{N}$ amortized over its life span. Let the arbitrage price be defined by

$$\pi_\delta := \pi_h - \pi_\ell \tag{1}$$

and define the arbitrage constant γ_i as follows:

$$\gamma_i := \frac{\pi_\delta - \pi_i}{\pi_\delta} \tag{2}$$

In order to have a viable arbitrage opportunity, we need

$$\pi_i \leq \pi_\delta \tag{3}$$

which corresponds to $\gamma_i \in [0, 1]$. The consumers discharge their storage during peak hours and charge them during off-peak hours.

The daily cost of energy consumption of a consumer $i \in \mathcal{N}$ for the peak period consumption $\mathbf{x}_i$ depends on the capacity investment C_i and is given by

$$J(\mathbf{x}_i, C_i) = \pi_i C_i + \pi_h (\mathbf{x}_i - C_i)^+ + \pi_\ell \min\{C_i, \mathbf{x}_i\}, \tag{4}$$

where $\pi_i C_i$ is the capital cost of acquiring C_i units of storage capacity, $\pi_h(\mathbf{x}_i - C_i)^+$ is the daily cost of the electricity purchase during peak price period, and $\pi_\ell \min\{\mathbf{x}_i, C_i\}$ is the daily cost of the electricity purchase during off-peak period to be stored for consumption during the peak period. We ignore the off-peak period electricity consumption of the consumer from the expression of J as its expression is independent of the storage capacity. The daily peak consumption of electricity is not known in advance and we assume it to be a random variable. Let F be the joint cumulative distribution function (CDF) of the collection of random variables $\{\mathbf{x}_i : i \in \mathcal{N}\}$ that represents the consumptions of the consumers in $\mathcal{N}$. The

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actual values of the daily peak consumptions are considered as independent and identically distributed observations of the random variables $\{\mathbf{x}_i : i \in \mathcal{N}\}$.

If $\mathcal{S} \subseteq \mathcal{N}$ is a subset of consumers, then $\mathbf{x}_\mathcal{S}$ denotes the aggregated peak consumption of $\mathcal{S}$ and its time average CDF is $F_\mathcal{S}$.

The daily cost of storage of a group of consumers $\mathcal{S} \subseteq \mathcal{N}$ with aggregated peak consumption $\mathbf{x}_\mathcal{S} = \sum_{i \in \mathcal{S}} \mathbf{x}_i$ and joint storage capacity $C_\mathcal{S}$ is

$$J(\mathbf{x}_\mathcal{S}, C_\mathcal{S}) = \pi_\mathcal{S} C_\mathcal{S} + \pi_h (\mathbf{x}_\mathcal{S} - C_\mathcal{S})^+ + \pi_\ell \min\{C_\mathcal{S}, \mathbf{x}_\mathcal{S}\} \tag{5}$$

where π_S is the daily capital cost of aggregated storage of the group amortized during its life span. Note that the individual storage costs (4) are obtained from (5) for the singleton sets $\mathcal{S} = \{i\}$.

The daily cost of storage given by (4) and (5) are random variables with expected values

$$J_\mathcal{S}(C_\mathcal{S}) = \mathbb{E} J_\mathcal{S}(\mathbf{x}_\mathcal{S}, C_\mathcal{S}), \quad \mathcal{S} \subseteq \mathcal{N}. \tag{6}$$

In the sequel, we will distinguish between the random variables and their realized values by using bold face fonts $\mathbf{x}_\mathcal{S}$ for the random variables and normal fonts $x_\mathcal{S}$ for their realized values.

Remark 1: For simplicity of the system model presentation, we assume that the random variables modeling the daily peak consumptions do not change with time. However, there exists empirical evidence that electricity consumption exhibits daily, weekly and seasonal periodical patterns [24], [25]. In order to consider time-varying random variables, the daily cost of the consumption during the peak period is obtained by averaging over the lifetime of the electricity storage. It is not difficult to prove that the expression of the daily cost (4) can still be applied when the distributions of the random variables change with time. In that case, the CDF of each random variable in the set $\{\mathbf{x}_i : i \in \mathcal{N}\}$ is the average of the time-varying CDFs over the lifetime of the electrical storage. The details are given in the Appendix.

Remark 2: A similar system model and cost set-up have been previously proposed in [14]. However, in that paper, the firms owning storage capacity behave strategically. In order to promote efficiency and coordination between the agents, a spot market for electricity is introduced where the storage investment decision of the firms impact the price. Under that scenario, each firm invests for its own storage strategically. It is shown that the non-cooperative game played among them has no Nash equilibrium. The strategic storage firms then align their investments and as a result they reach a Nash equilibrium which also maximizes social welfare. By contrast, in our model, we assume that the consumers are not strategic, rather they fully cooperate. Consequently, we do not require a spot market for agent coordination. We also do not require an aggregator or any additional communication mechanisms.

C. Quantifying the Benefit of Cooperation

We are interested in studying and quantifying the benefit of cooperation in the two scenarios. In the first scenario, the consumers already have installed storage capacity $\{C_i : i \in \mathcal{N}\}$ that they acquired in the past. Each of the consumers can have a different storage technology that was acquired at a different time compared to the other consumers. Consequently, each consumer has a different daily capital cost π_i. The consumers aggregate their storage capacities and they operate using the same strategy, they use the aggregated storage capacity to store energy during off-peak periods that they will later use during peak periods. By aggregating storage devices, the unused capacity of some consumers is used by others producing cost savings for the group. We analyze this scenario using cooperative game theory and develop an efficient allocation rule of the daily storage cost that is satisfactory for every consumer.

In the second scenario, we consider a group of consumers that join to buy storage capacity that they want to use in a cooperative way. First, the group of consumers have to make a decision about how much storage capacity they need to acquire and then they have to share the expected cost among the group participants. The decision problem is modeled as an optimization problem where the group of consumers minimize the expected cost of daily storage. The problem of sharing the expected cost is modeled using cooperative game theory. We quantify the reduction in the expected cost of storage for the group and develop a mechanism to allocate the expected cost among the participants that is satisfactory for all of them.

III. Background: Coalitional Game Theory for Cost Sharing

Game theory deals with rational behavior of economic agents in a mutually interactive setting [26]. Broadly speaking, there are two major categories of games: non-cooperative games and cooperative games. Cooperative games (or coalitional games) have been used extensively in diverse disciplines such as social science, economics, philosophy, psychology and communication networks [15], [27]. Here, we focus on cooperative games for cost sharing [28].

Let $\mathcal{N} := \{1, 2, \ldots, N\}$ denote a finite collection of players. In a cooperative game for cost sharing, the players want to minimize their joint cost and share the resulting cost cooperatively.

Definition 1 (Coalition): A *coalition* is any subset $\mathcal{S} \subseteq \mathcal{N}$. The number of players in a coalition $\mathcal{S}$ is denoted by its cardinality, $|\mathcal{S}|$. The *set of all possible coalitions* is defined as the power set $2^\mathcal{N}$ of $\mathcal{N}$. The *grand coalition* is the set of all players, $\mathcal{N}$.

Definition 2 (Game and Value): A cooperative game is defined by a pair $(\mathcal{N}, v)$ where $v : 2^\mathcal{N} \to \mathbb{R}$ is the *value function* that assigns a real value to each coalition $\mathcal{S} \subseteq \mathcal{N}$. Hence, the *value of coalition* $\mathcal{S}$ is given by $v(\mathcal{S})$. For the cost sharing game, $v(\mathcal{S})$ is the total cost of the coalition.

Definition 3 (Subadditive Game): A cooperative game $(\mathcal{N}, v)$ is *subadditive* if, for any pair of disjoint coalitions $\mathcal{S}, \mathcal{T} \subset \mathcal{N}$ with $\mathcal{S} \cap \mathcal{T} = \emptyset$, we have $v(\mathcal{S}) + v(\mathcal{T}) \geq v(\mathcal{S} \cup \mathcal{T})$.

Here we consider the value of the coalition $v(\mathcal{S})$ is *transferable* among players. The central question for a subadditive cost sharing game with transferrable value is how to *fairly* distribute the coalition value among the coalition members.

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Definition 4 (Cost Allocation): A *cost allocation* for the coalition $\mathcal{S} \subseteq \mathcal{N}$ is a vector $\xi \in \mathbb{R}^N$ whose entry ξ_i represents the allocation to member $i \in \mathcal{S}$ ($\xi_i = 0, \quad i \notin \mathcal{S}$).
For any coalition $\mathcal{S} \subseteq \mathcal{N}$, let $\xi_{\mathcal{S}}$ denote the sum of cost allocations for every coalition member, *i.e.* $\xi_{\mathcal{S}} = \sum_{i\in\mathcal{S}} \xi_i$.

Definition 5 (Imputation): A cost allocation ξ for the grand coalition $\mathcal{N}$ is said to be an *imputation* if it is simultaneously efficient –*i.e.* $v(\mathcal{N}) = \xi_{\mathcal{N}}$, and individually rational –*i.e.* $v(i) \geq \xi_i, \forall i \in \mathcal{N}$. Let $\mathcal{I}$ denote the set of all imputations.

The fundamental solution concept for cooperative games is the *core* [26].

Definition 6 (The Core): The *core* $\mathcal{C}$ for the cooperative game $(\mathcal{N}, v)$ with *transferable cost* is defined as the set of cost allocations such that no coalition can have cost which is lower than the sum of current costs of all the members under the given allocation.

$$\mathcal{C} := \left\{ \xi \in \mathcal{I} : v(\mathcal{S}) \geq \xi_{\mathcal{S}}, \forall \mathcal{S} \in 2^{\mathcal{N}} \right\}. \tag{7}$$

The existence of a nonempty core guarantees the stability of the grand coalition, because no subset of players obtains additional benefit by defecting from the grand coalition and forming a separate coalition. Thus, the core is a solution concept for cooperative games analogous to the Nash equilibrium for noncooperative games [26]. Moreover, any imputation in the core allocates the cost of the grand coalition in a stable way among the game players. Studying the existence of a nonempty core is not a trivial task, because it requires solving a linear feasibility problem (7) with a number of constraints that increases exponentially with the number of players.

A classical result in cooperative game theory, known as the Bondareva-Shapley theorem, gives a necessary and sufficient condition for a game to have nonempty core. To state this theorem, we need the following definition.

Definition 7 (Balanced Game and Balanced Map): A cooperative game $(\mathcal{N}, v)$ for cost sharing is *balanced* if for any balanced map α, $\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S}) v(\mathcal{S}) \geq v(\mathcal{N})$ where the map $\alpha : 2^{\mathcal{N}} \rightarrow [0, 1]$ is said to be *balanced* if for all $i \in \mathcal{N}$, we have $\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S}) \mathbf{1}_{\mathcal{S}}(i) = 1$, where $\mathbf{1}_{\mathcal{S}}$ is the indicator function of the set $\mathcal{S}$, *i.e.* $\mathbf{1}_{\mathcal{S}}(i) = 1$ if $i \in \mathcal{S}$ and $\mathbf{1}_{\mathcal{S}}(i) = 0$ if $i \notin \mathcal{S}$.
Next we state the Bondareva-Shapley theorem.

Theorem 1 (Bondareva-Shapley Theorem [15]): A coalitional game has a nonempty core if and only if it is balanced.

For a cooperative game with non-empty core, the total cost of the grand coalition can be allocated in a stabilizing manner. In that case, no individual or coalition can reduce their allocated cost by defecting from the grand coalition.

There are many approaches to finding solutions for a cooperative game. If a game is balanced, the nucleolus [27] and worst case excess [17] are solutions that are always in the core. For a concave cost sharing game, the Shapley value [29] is in the core.

IV. Main Results

A. Scenario I: Realized Cost Minimization with Already Procured Storage Elements

Our first concern is to study if there is some benefit in cooperation of the consumers by sharing the storage capacity that they already have. To analyze this scenario we shall formulate our problem as a coalitional game.

1) Coalitional Game and Its Properties: The players of the cooperative game are the consumers that share their storage and want to reduce their realized joint storage investment cost. For any coalition $\mathcal{S} \subseteq \mathcal{N}$, the cost of the coalition is $u(\mathcal{S})$ which is the realized cost of the joint storage investment $C_{\mathcal{S}} = \sum_{i\in\mathcal{S}} C_i$. Each consumer may have a different daily capital cost of storage $\{\pi_i : i \in \mathcal{N}\}$, because they did not necessarily purchase their storage systems at the same time or at the same price for KW. The realized cost of the joint storage for the peak period consumption $x_{\mathcal{S}} = \sum_{i\in\mathcal{S}} x_i$ is given by

$$u(\mathcal{S}) = J(x_{\mathcal{S}}, C_{\mathcal{S}}) \tag{8}$$

where J was defined in (5). Since we are using the realized value of the aggregated peak consumption $x_{\mathcal{S}}$, $J(x_{\mathcal{S}}, C_{\mathcal{S}})$ is not longer a random variable.

In order to show that cooperation is advantageous for the members of the group, we have to prove that the game is subadditive. In such a case, the joint daily investment cost of the consumers is never greater that the sum of the individual daily investment costs. Subadditivity of the cost sharing coalitional game is established in Theorem 2.

Theorem 2: The cooperative game for storage investment cost sharing $(\mathcal{N}, u)$ with the cost function u defined in (8) is subadditive.

Proof: See the Appendix.

However, subadditivity is not enough to provide satisfaction of the coalition members. We need a stabilizing allocation mechanism of the aggregated cost. Under a stabilizing cost sharing mechanism no member in the coalition is impelled to break up the coalition. Such a mechanism exists if the cost sharing coalitional game is balanced. Balancedness of the cost sharing coalitional game is established in Theorem 3.

Theorem 3: The cooperative game for storage investment cost sharing $(\mathcal{N}, u)$ with the cost function u defined in (8) is balanced.

Proof: See the Appendix.

2) Sharing of Realized Cost: Since the cost sharing cooperative game $(\mathcal{N}, u)$ is balanced, its core is nonempty and there always exist cost allocations that stabilize the grand coalition. One of this coalitions is the nucleolus while another one is the allocation that minimizes the worst case excess [17]. However, computing these allocations requires solving linear programs with a number of constraints that grows exponentially with the cardinality of the grand coalition. There have been improvements in computation of nucleolus in recent years [30]. However, in our case, it is possible to develop an analytical formula for cost allocation that is in the core. Here the realized joint cost of the grand coalition can be written as

$$u(\mathcal{N}) = \begin{cases} \pi_i C_{\mathcal{N}} + \pi_h (x_{\mathcal{N}} - C_{\mathcal{N}}) + \pi_\ell C_{\mathcal{N}}, & \text{if } x_{\mathcal{N}} \geq C_{\mathcal{N}} \\ \pi_i C_{\mathcal{N}} + \pi_\ell x_{\mathcal{N}}, & \text{if } x_{\mathcal{N}} < C_{\mathcal{N}} \end{cases}$$

The terms $C_{\mathcal{N}}$ and $x_{\mathcal{N}}$ can be split up among individual agents and as a result we get the following allocation.

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Allocation 1 (Realized Cost Allocation for Scenario I): Define the cost allocation $\{\xi_i : i \in \mathcal{N}\}$ as follows:

$$\xi_i := \begin{cases} \pi_i C_i + \pi_h(x_i - C_i) + \pi_\ell C_i, & \text{if } x_{\mathcal{N}} \geq C_{\mathcal{N}} \\ \pi_i C_i + \pi_\ell x_i, & \text{if } x_{\mathcal{N}} < C_{\mathcal{N}} \end{cases} \quad (9)$$

for all $i \in \mathcal{N}$.

We establish in Theorem 4, this cost allocation belongs to the core of the cost sharing cooperative game.

Theorem 4: The cost allocation $\{\xi_i : i \in \mathcal{N}\}$ defined in Allocation 1 belongs to the core of the cost sharing cooperative game $(\mathcal{N}, u)$.

Proof: See the Appendix.

Unlike the nucleolus or the cost allocation minimizing the worst-case excess, Allocation 1 has an analytical expression and can be easily obtained without any costly computation. Thus, we have developed a strategy such that consumers that independently invested in storage, and are subject to a two period (peak and off-peak) TOU pricing mechanism can reduce their costs by sharing their storage devices. Moreover, we have proposed a cost sharing allocation rule that stabilizes the grand coalition. This strategy can be considered a weak cooperation because each consumer acquired its storage capacity independently of each other, but they agree to share the joint storage capacity.

In the next section we consider a stronger cooperation problem, where a group of consumers decide to invest jointly in storage capacity.

B. Scenario II: Expected Cost Minimization for Joint Storage Investment

In this scenario, we consider a group of consumers indexed by $i \in \mathcal{N}$, that decide to jointly invest in storage capacity. The consumers in this scenario cooperate with each other with a long-term profit goal. We are interested in studying whether cooperation provides a benefit for the coalition members for the long term.

1) Coalitional Game and Its Properties: Similar to the previous case, only the peak consumption is relevant in the investment decision. Let $\mathbf{x}_i$ denote the daily peak period consumption of consumer $i \in \mathcal{N}$. Unlike the previous scenario, here $\mathbf{x}_i$ is a random variable with marginal cumulative distribution function (CDF) F_i. The daily cost of the consumer $i \in \mathcal{N}$ depends on the storage capacity investment of the consumer as per (4). This cost is also a random variable. If the consumer is risk neutral, it acquires the storage capacity C_i^* that minimizes the expected value of the daily cost

$$C_i^* = \arg\min_{C_i \geq 0} J_i(C_i), \quad (10)$$

where

$$J_i(C_i) = \mathbb{E}J(\mathbf{x}_i, C_i), \quad (11)$$

and $\pi_{\mathcal{S}}$ is the daily capital cost of storage amortized over its lifespan that in this case is the same for each of the consumers –*i.e.* $\pi_i = \pi_{\mathcal{S}}$ for all $i \in \mathcal{N}$, because we assume that they buy storage devices of the same technology at the same time. This problem has been previously solved in [14] and its solution is given by Theorem 5.

Theorem 5 ([14]): The storage capacity of a consumer $i \in \mathcal{N}$ that minimizes its daily expected cost is C_i^*, where

$$F_i(C_i^*) = \frac{\pi_\delta - \pi_{\mathcal{S}}}{\pi_\delta} = \gamma_{\mathcal{S}}$$

and the resulting optimal cost is

$$J_i^* = J_i(C_i^*) = \pi_\ell \mathbb{E}[\mathbf{x}_i] + \pi_{\mathcal{S}} \mathbb{E}[\mathbf{x}_i \mid \mathbf{x}_i \geq C_i^*]. \quad (12)$$

Let us consider a group of consumers $\mathcal{S} \subseteq N$ that decide to invest in joint storage capacity. The peak consumption of the coalition is $\mathbf{x}_{\mathcal{S}} = \sum_{i\in\mathcal{S}} \mathbf{x}_i$ with CDF $F_{\mathcal{S}}$. We also assume that the joint CDF of all the agent's peak consumptions F is known or can be estimated from historical data. By applying Theorem 5, the optimal investment in storage capacity of the coalition $\mathcal{S} \subseteq \mathcal{N}$ is $C_{\mathcal{S}}^*$ such that $F_{\mathcal{S}}(C_{\mathcal{S}}^*) = \gamma_{\mathcal{S}}$ and the optimal cost is

$$J_{\mathcal{S}}^* = J_{\mathcal{S}}(C_{\mathcal{S}}^*) = \pi_\ell \mathbb{E}[\mathbf{x}_{\mathcal{S}}] + \pi_{\mathcal{S}} \mathbb{E}[\mathbf{x}_{\mathcal{S}} \mid \mathbf{x}_{\mathcal{S}} \geq C_{\mathcal{S}}^*]. \quad (13)$$

Consider the cost sharing cooperative game $(\mathcal{N}, v)$ where the cost function $v : 2^{\mathcal{N}} \to \mathbb{R}$ is defined as follows

$$v(\mathcal{S}) = J_{\mathcal{S}}^* = \arg\min_{C_{\mathcal{S}} \geq 0} J_{\mathcal{S}}(C_{\mathcal{S}}), \quad (14)$$

where $J_{\mathcal{S}}^*$ was defined in (13).

Similar to the case of consumers that already own storage capacity and decide to cooperate to reduce their costs, here we prove that the cooperative game is subadditive so that the consumer obtain a reduction of cost. This is the result in Theorem 6.

Theorem 6: The cooperative game for storage investment cost sharing $(\mathcal{N}, v)$ with the cost function v defined in (14) is subadditive.

Proof: See the Appendix.

We also need a cost allocation rule that is stabilizing. Theorem 7 establishes that the game is balanced and has a stabilizing allocation.

Theorem 7: The cooperative game for storage investment cost sharing $(\mathcal{N}, v)$ with the cost function v defined in (14) is balanced.

Proof: See the Appendix.

2) Stable Sharing of Expected Cost: Similar to the previous scenario, we are able to develop a cost allocation rule that is in the core. The expected cost of the grand coalition for the aggregated daily peak consumption is

$$u(\mathcal{N}) = \pi_\ell \mathbb{E}[\sum_{i\in\mathcal{N}} \mathbf{x}_i] + \pi_{\mathcal{N}} \mathbb{E}[\sum_{i\in\mathcal{N}} \mathbf{x}_i \mid \mathbf{x}_{\mathcal{N}} \geq C_{\mathcal{N}}^*]$$

We need to split up the joint cost among the individuals and we end up having an allocation with nice properties.

The resultant cost allocation rule has an analytical formula and can be efficiently computed. This allocation rule is defined as follows.

Allocation 2 (Expected Cost Allocation for Scenario II): Define the cost allocation $\{\zeta_i : i \in \mathcal{N}\}$ as follows:

$$\zeta_i := \pi_\ell \mathbb{E}[\mathbf{x}_i] + \pi_{\mathcal{N}} \mathbb{E}[\mathbf{x}_i \mid \mathbf{x}_{\mathcal{N}} \geq C_{\mathcal{N}}^*], \ i \in \mathcal{N}. \quad (15)$$

In the following theorem, we prove that Allocation 2 provides a sharing mechanism of the expected daily storage cost of a coalition of agents that is in the core of the cooperative game.

Theorem 8: The cost allocation $\{\zeta_i : i \in \mathcal{N}\}$ defined in Allocation 2 belongs to the core of the cost sharing cooperative game $(\mathcal{N}, v)$.

Proof: See the Appendix.

The expression of cost allocation $\{\zeta_i : i \in \mathcal{N}\}$ is similar to the strong Nash equilibrium storage investment obtained in [14] for a group of strategic agents that invests in individual storage and trade the excess stored energy through a spot market. However, since our storage sharing set-up is different, we do not need to implement a spot market and no alignment condition is required.

3) Sharing of Realized Cost: Based on the above results, the consumers can invest on joint storage and they will make savings for long term. But the cost allocation ζ_i defined by (15) is in expectation. The realized allocation will be different due to the randomness of the daily consumption. A cost allocation of the realized cost similar to Allocation 1 cannot be applied in this case because the storage capacity $C^*_{\mathcal{N}}$ belongs to the grand coalition and cannot be easily split among the participants. Also in this case, as the consumers are having a common storage, they are interested in having cost savings in the long-term and it is not imperative for them to have a short-term cost allocation in the core like in Scenario I. It is enough to develop a cost allocation of the realized cost that is consistent with the expected allocation in the long-term. Next, we propose a daily cost allocation for the k-th day.

Allocation 3 (Realized Cost Allocation for Scenario II): Define the cost allocation $\{\rho_i^k : i \in \mathcal{N}\}$ as follows:

$$\rho_i^k = \beta_i \pi_{\mathcal{N}}^k, \tag{16}$$

where superscript k represents the day, $\pi_{\mathcal{N}}^k$ is the realized cost for the grand coalition on the k-th day and $\beta_i = \frac{\zeta_i}{\sum_{i=1}^N \zeta_i}$.

Since $\sum_{i=1}^N \beta_i = 1$, $\sum_{i=1}^N \rho_i^k = \pi_{\mathcal{N}}^k$ and the cost allocation is budget balanced. Also using strong law of large numbers, the average cost allocation for K days $\bar{\rho}_i(K)$ given by

$$\bar{\rho}_i(K) = \frac{1}{K}\sum_{k=1}^{K} \rho_i^k \tag{17}$$

approaches ζ_i as $K \to \infty$, and the realized allocation is strongly consistent with the allocation of the expected cost $\{\zeta_i : i \in \mathcal{N}\}$.

V. Benefit of Cooperation

A. Scenario I

The benefit of cooperation by joint operation of storage reflected in the total reduction of cost is given by

$$\sum_{i\in\mathcal{S}} J_i - J_{\mathcal{S}} = \pi_h\Big(\sum_{i\in\mathcal{S}}(x_i - C_i)^+ - (x_{\mathcal{S}} - C_{\mathcal{S}})^+\Big) + \pi_\ell\Big(\sum_{i\in\mathcal{S}}\min\{C_i, x_i\} - \min\{C_{\mathcal{S}}, x_{\mathcal{S}}\}\Big), \tag{18}$$

where the reduction for individual agent with cost allocation (9) is

$$J_i - \zeta_i := \begin{cases} \pi_\delta(C_i - x_i)^+, & \text{if } x_{\mathcal{N}} \geq C_{\mathcal{N}} \\ \pi_\delta(x_i - C_i)^+, & \text{if } x_{\mathcal{N}} < C_{\mathcal{N}} \end{cases} \tag{19}$$

B. Scenario II

The benefit of cooperation given by the reduction in the expected cost that the coalition $\mathcal{S}$ obtains by jointly acquiring and exploiting the storage is

$$\sum_{i\in\mathcal{S}} J_i^* - J_{\mathcal{S}}^* = \pi_{\mathcal{S}}\sum_{i\in\mathcal{S}}\mathbb{E}[\mathbf{x}_i \mid \mathbf{x}_i \geq C_i^*] - \pi_{\mathcal{S}}\mathbb{E}[\mathbf{x}_{\mathcal{S}} \mid \mathbf{x}_{\mathcal{S}} \geq C_{\mathcal{S}}^*], \tag{20}$$

and the reduction in expected cost of each participant assuming that the expected cost of the coalition is split using cost allocation (15) is

$$J_i^* - \zeta_i = \pi_{\mathcal{S}}\mathbb{E}[\mathbf{x}_i \mid \mathbf{x}_i \geq C_i^*] - \pi_{\mathcal{S}}\mathbb{E}[\mathbf{x}_i \mid \mathbf{x}_{\mathcal{S}} \geq C_{\mathcal{S}}^*]. \tag{21}$$

VI. Potential Model Extensions

Our model has been designed to be simple enough to be mathematically tractable for analysis of the effects of aggregation. We have ignored aspects as complex pricing mechanisms, storage efficiency or capacity constraints.

In reality, there can be more than two time periods. The time-of-use pricing can be extended to more than two periods. Consider a TOU pricing mechanism with p periods and prices $\{\pi_{t_k} : k = 1, \ldots, p\}$ such that $\pi_{t_k} < \pi_{t_\ell}$ if $k < \ell$. The daily cost of storage of a coalition $\mathcal{S} \subseteq \mathcal{N}$ with consumptions $\mathbf{x}_{\mathcal{S}} = \{\mathbf{x}_{\mathcal{S}}(t_k) : k = 1, \ldots, p\}$ is given by

$$J(\mathbf{x}_{\mathcal{S}}, C_{\mathcal{S}}) = \pi_{\mathcal{S}} C_{\mathcal{S}} + \pi_{t_0}\min\left\{\sum_{k=1}^{p}\mathbf{x}_{\mathcal{S}}(t_k), C_{\mathcal{S}}\right\} + \sum_{k=1}^{p}\pi_{t_k}\min\left\{\mathbf{x}_{\mathcal{S}}(t_k), \left(\sum_{\ell=k}^{p}\mathbf{x}_{\mathcal{S}}(t_k) - C_{\mathbf{S}}\right)^+\right\} \tag{22}$$

If the joint probability distribution functions of the consumption random variables $\mathbf{x}_{\mathcal{S}}(t_k)$ are known or can be estimated, then the optimal storage investments can be obtained by solving an optimization program

$$C_{\mathcal{S}}^* = \arg\min_{C_{\mathcal{S}}\geq 0}\mathbb{E}[J(\mathbf{x}_{\mathcal{S}}, C_{\mathcal{S}})] \tag{23}$$

An analysis of cooperation under this multiperiod pricing scheme could be explored in the future using this setting.

Our model considers ideal storage. However, in reality storage can degrade. Charging and discharging efficiency can be incorporated to the model and these have the effect of inflating the off-peak price and reducing the peak price respectively. Since the degradation can happen with or without sharing, we believe that its impact on the nature of our conclusions is likely to be be minimal. Also we assumed in our model that our infrastructure has the necessary capacity. This scenario can be approached using some inequality constraints along with the objective function to handle capacity or network constraints. In the future, our work can be generalized in these important directions.

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TABLE I
CODES OF THE PECAN ST. HOUSEHOLDS

Case Study Index	1	2	3	4	5
Pecan St. Code	26	624	2945	5218	5658

VII. CASE STUDY

We present a case study to illustrate our results. For this case study, we use data from the Pecan St project [31]. We consider a two-period ToU tariff with $\pi_h = 55$¢/KWh, and $\pi_\ell = 20$¢/KWh. Electricity storage is currently expensive. The amortized cost of Tesla's Powerwall Lithium-ion battery is around 25¢/KWh per day. But storage prize is projected to reduce by 30% by 2020 [32]. Keeping in mind this projection, we consider $\pi_S = 15$¢/KWh.

A group of five households decide to cooperate with their existing storage systems or acquire new storage. The Pecan St. codes of the five households used in this study are given in Table I. The historical realized consumption data of this group of households during the year 2016 have been retrieved and used for the study. Using these historical data, we conduct a statistical study of the peak consumptions for the five households. Peak consumption period in Texas corresponds to non-holidays and non-weekends from 7h to 23h. We first remove the days and periods corresponding to off-peak consumption. In Table II, we show the statistical summary of the peak consumptions for the five households. The summary includes mean, standard deviation, minimum and maximum, and three quartiles (25%, 50% and 75%). We observe that on average, the largest daily peak consumption corresponds to household 5 with almost 30 KWh, followed by household 1, with 22 KWh. The remaining households have similar average daily peak consumptions between 13 and 14 KWh. Regarding the variability in consumption, household 5 has the larger variation, followed by households 1 and 3, those have a very similar variation, while household 5 has the lower variation in daily peak consumption. The statistical summary of the daily peak consumption is also graphically presented using box plots in Figure 2.

Considering that the real peak consumptions data across the different days are independent observations of random variables, we estimate the the CDFs of these random variables for each consumer $\{\mathbf{x}_i : i \in \mathcal{N}\}$ and for the grand coalition $\mathbf{x}_{\mathcal{N}} = \sum_{i\in\mathcal{N}} \mathbf{x}_i$. The estimated CDFs are depicted in Figure 3. The correlation coefficients of the peak consumptions of these five households are given in Table III. The peak consumption distributions are not completely dependent, as we can check from Figure 3 and Table III.

It means that there is room for reduction in cost by making a coalition. The optimal investments in storage for the five households and for the grand coalition are given in Table IV.

We first assume that the five households buy storage independently and then decide to cooperate by sharing their storage to reduce the realized cost. This corresponds to Scenario I. For simplicity of computation and comparison with Scenario II, we consider $\pi_i = \pi_S$ for all i. The realized cost is allocated using (9). In Table V, we show the allocation of the realized

TABLE II
STATISTICAL SUMMARY OF THE PEAK CONSUMPTIONS (IN KWH)

Household	1	2	3	4	5
Mean	22.0648	13.8197	13.6311	13.1474	28.9748
St. Dev.	8.7048	6.9377	8.9827	4.7216	11.7795
Min	6.0059	5.0604	3.4408	3.6397	8.3999
Q25	14.5427	8.2933	6.4359	9.9940	19.2315
Q50	20.5878	12.3185	11.7135	12.6130	26.0138
Q75	28.2698	17.8733	18.3429	16.4175	37.7488
Max	46.4165	40.0662	41.3888	30.0907	61.5894

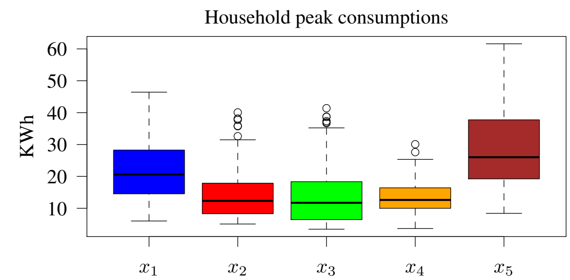

Fig. 2. Boxplots of the peak consumptions of the five households

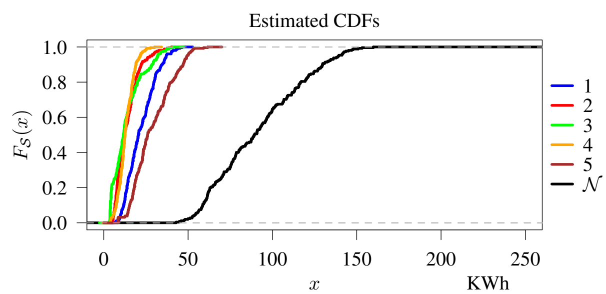

Fig. 3. Estimated CDFs of the peak consumption of the five households and their aggregated consumption

TABLE III
CORRELATION COEFFICIENTS OF PEAK CONSUMPTIONS FOR THE FIVE HOUSEHOLDS

	1	2	3	4	5
1	1.000000	0.363586	0.297733	0.292073	0.486665
2	0.363586	1.000000	0.132320	0.453056	0.157210
3	0.297733	0.132320	1.000000	0.085868	0.365212
4	0.292073	0.453056	0.085869	1.000000	-0.056696
5	0.486665	0.157210	0.365212	-0.056696	1.000000

TABLE IV
OPTIMAL STORAGE CAPACITY INVESTMENTS (IN KWH) AND MINIMAL EXPECTED DAILY COST (IN \$) FOR THE INDIVIDUAL HOUSEHOLDS

C_1^*	C_2^*	C_3^*	C_4^*	C_5^*	$\sum_{i\in\mathcal{N}} C_i^*$
22.98	14.09	12.64	13.21	29.82	92.74
J_1^*	J_2^*	J_3^*	J_4^*	J_5^*	$\sum_{i\in\mathcal{N}} J_i^*$
9.00	5.80	6.01	5.26	11.89	37.96

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TABLE V
ALLOCATION OF THE REALIZED COST FOR SCENARIO I FOR THE FIRST TEN DAYS OF THE YEAR (IN $)

Day	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5
1	4.93	6.13	4.37	5.50	9.05
2	4.65	6.25	3.44	5.67	9.47
3	5.41	4.83	3.00	5.41	8.20
4	6.76	3.74	3.78	4.18	7.34
5	7.61	4.03	4.06	3.72	7.99
6	6.46	5.17	4.05	5.73	8.13
7	6.54	7.61	3.88	5.37	7.97
8	5.84	4.11	5.33	4.56	8.32
9	6.40	3.94	4.83	4.83	7.87
10	6.04	4.46	4.75	3.10	7.92

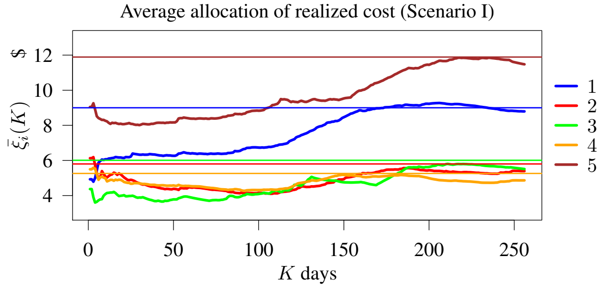

Fig. 4. Evolution of the average allocation of the realized cost in Scenario I with the number of days

aggregated cost for the ten first days of 2016, assuming that the households have storage capacities $\{C_i^* : i \in \mathcal{N}\}$.

In Figure 4, we depict the evolution of the average allocation of the realized cost of storage to each household for the year 2016. The average allocation for K days is given by

$$\bar{\xi}_i(K) = \frac{1}{K}\sum_{k=1}^{K} \xi_i^k, \ i \in \mathcal{N}, \tag{24}$$

where superscript k represent the day, and K is the number of days. The average cost allocation is compared to the optimal expected costs J_i^*. The peak consumptions of the household i for days $k = 1, \ldots, K$ are observations that are independent across the days and identically distributed and so are the realized cost allocations. By the Strong Law of Large Numbers, the average value of the realized allocation converge *almost surely* to the expected value of the cost allocation and satisfies $\xi_i^\infty = \lim_{K\to\infty} \bar{\xi}_i(K) \le J_i^*$ for $i \in \mathcal{N}$, as it is shown in Figure 4.

Next, we consider the case that the consumers jointly decide to buy a common energy storage. In Table VI, we show the optimal storage capacity investment of the grand coalition and the corresponding minimum expected daily cost of electricity consumption. We also show in this table the allocation of the expected daily storage cost given by (15). Note that the optimal storage capacity for the grand coalition $C_\mathcal{N}^* =$ in this particular case is slightly larger than the sum of the optimal individual storage capacities $\sum_{i\in\mathcal{N}} C_i^* =$. The reduction in cost for the consumers coalition is about 5%, however those with less correlation with the other, have a larger reduction. Consumers 3 and 4 have cost reductions higher than 7%, while

TABLE VI
OPTIMAL STORAGE CAPACITY INVESTMENT (IN KWH), MINIMAL EXPECTED DAILY COST (IN $) AND ITS ALLOCATION (IN $) FOR THE GRAND COALITION

$C_\mathcal{N}^*$	$J_\mathcal{N}^*$	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5
95.58	36.04	8.82	5.43	5.50	4.88	11.40

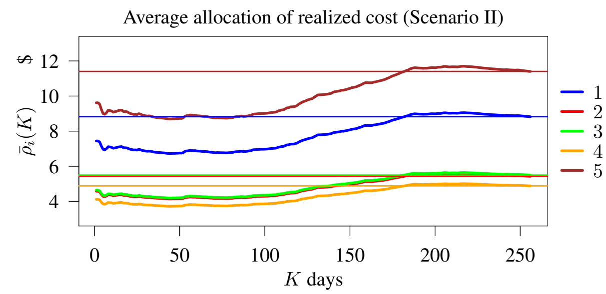

Fig. 5. Evolution of the average allocation of the realized cost in Scenario II with the number of days

Consumer 1, whose consumption is more correlated with the other, have about 2.4% of cost reduction.

We complete the study by applying Allocation 3 to the realized values of the peak consumptions for the complete year 2016. The allocation of the realized cost is strongly consistent with the allocation of the expected cost and converge on average *almost surely* to $\{\zeta_i : i \in \mathcal{N}\}$. Finally, we compare the average allocation of storage cost in both scenarios for the year 2016 using Allocations 1 and 2. As expected, Scenario II obtains the minimum average daily cost, however the difference with the average daily cost in Scenario I is very small. So in this case, there is not much additional benefit by having a common storage and the consumers can obtain satisfactory cost reductions by implementing only weak cooperation. It is expected that aggregation of a larger number of households with a large diversity in peak consumption would obtain greater benefit with Scenario II.

VIII. CONCLUSIONS

In this paper, we explored sharing opportunities of electricity storage elements among a group of consumers. We used cooperative game theory as a tool for modeling. Our results prove that cooperation is beneficial for agents that either already have storage capacity or want to acquire storage capacity. In the first scenario, the different agents only need the infrastructure to share their storage devices. In such a case the operative scheme is really simple, because each agent only has to storage at off-peak periods as much as possible energy

TABLE VII
COMPARATIVE STUDY OF AVERAGE COST ALLOCATIONS (IN $) FOR SCENARIO I AND SCENARIO II DURING 2016

1	2	3	4	5	Sum
8.79	5.40	5.51	4.87	11.48	36.07
8.82	5.43	5.50	4.88	11.40	36.05

that they will consume during peak periods. At the end of the day, the realized cost is shared among the participants. In the second scenario, the coalition members can take an optimal decision about how much capacity they jointly acquire by minimizing the expected daily storage cost. We showed that the cooperative games in both the cases are balanced. We also developed allocation rules with analytical formulas in both the cases. Thus, our results suggest that sharing of storage in a cooperative way is very much useful for all the agents and the society.

As future work, we are extending our results to a more realistic set-up including multi-period pricing mechanisms, storage degradation cost and capacity constraints in the model. We also plan to analyze the impact of cooperation on the dynamic behavior of power systems.

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APPENDIX

A. System model for time-varying random variables

Let us denote the time-varying random variables modeling the peak period consumptions by $\{\mathbf{x}_i(t) : i \in \mathcal{N}\}$ with known CDFs $\{F_i(x;t) : i \in \mathcal{N}\}$. The cost of energy consumption of a consumer $i \in \mathcal{N}$ for the peak period consumption at any day $t \in \{1, \ldots, T\}$ of the lifetime of the electrical storage is given by

$$J(\mathbf{x}_i(t), C_i) = \pi_i C_i + \pi_h(\mathbf{x}_i(t) - C_i)^+ + \pi_\ell \min\{C_i, \mathbf{x}_i(t)\}.$$

The consumer is interested in minimizing the expected value

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of the daily cost over the lifetime of the electrical storage, *i.e.*

$$\begin{aligned}
&\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} J(\mathbf{x}_i(t), C_i)\right] \\
&= \pi_i C_i + \pi_h \int_0^{C_i} \frac{1}{T}\sum_{t=1}^{T}(x - C_i)dF_i(x;t) + \\
&\qquad \pi_\ell \int_0^{C_i} \frac{1}{T}\sum_{t=1}^{T} x dF_i(x;t) + \pi_\ell \int_{C_i}^{\infty} \frac{1}{T}\sum_{t=1}^{T} C_i dF_i(x;t) \\
&= \pi_i C_i + \pi_h \int_0^{C_i} (x - C_i)\frac{1}{T}\sum_{t=1}^{T} dF_i(x;t) + \\
&\qquad \pi_\ell \int_0^{C_i} x\frac{1}{T}\sum_{t=1}^{T} dF_i(x;t) + \pi_\ell \int_{C_i}^{\infty} C_i\frac{1}{T}\sum_{t=1}^{T} dF_i(x;t).
\end{aligned}$$

By introducing a stationary random variable $\mathbf{x}_i$ with CDF given by $F_i(x) = \frac{1}{T}\sum_{t=1} F_i(x;t)$, we obtain

$$\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} J(\mathbf{x}_i(t), C_i)\right] = \mathbb{E}\left[J(\mathbf{x}_i, C_i)\right],$$

where $J(\mathbf{x}_i, C_i)$ is given by (4).

B. Proof of Theorem 2

We shall prove that J defined by (4) is a subadditive function. For any nonnegative real numbers $x_\mathcal{S}$, $x_\mathcal{T}$, $C_\mathcal{S}$, $C_\mathcal{T}$, we define $J_\mathcal{S} = J(x_\mathcal{S}, C_\mathcal{S})$, $J_\mathcal{T} = J(x_\mathcal{T}, C_\mathcal{T})$, $J_{\mathcal{S}\cup\mathcal{T}} = J(x_\mathcal{S} + x_\mathcal{T}, C_\mathcal{S} + C_\mathcal{T})$, then

$$\begin{aligned}
J_\mathcal{S} &= \sum_{i\in\mathcal{S}} \pi_i C_i + \pi_h (x_\mathcal{S} - C_\mathcal{S})^+ + \pi_\ell \min\{C_\mathcal{S}, x_\mathcal{S}\}, \\
J_\mathcal{T} &= \sum_{i\in\mathcal{T}} \pi_i C_i + \pi_h (x_\mathcal{T} - C_\mathcal{T})^+ + \pi_\ell \min\{C_\mathcal{T}, x_\mathcal{T}\}, \\
J_{\mathcal{S}\cup\mathcal{T}} &= \sum_{i\in\mathcal{S}\cup\mathcal{T}} \pi_i C_i + \pi_h (x_\mathcal{S} + x_\mathcal{T} - C_\mathcal{S} - C_\mathcal{T})^+ + \\
&\qquad \pi_\ell\{C_\mathcal{S} + C_\mathcal{T}, x_\mathcal{S} + x_\mathcal{T}\}.
\end{aligned}$$

We can distinguish four cases[1]: (a) $x_\mathcal{S} \geq C_\mathcal{S}$ and $x_\mathcal{T} \geq C_\mathcal{T}$, (b) $x_\mathcal{S} \geq C_\mathcal{S}$, $x_\mathcal{T} < C_\mathcal{T}$ and $x_\mathcal{S} + x_\mathcal{T} \geq C_\mathcal{S} + C_\mathcal{T}$, (c) $x_\mathcal{S} \geq C_\mathcal{S}$, $x_\mathcal{T} < C_\mathcal{T}$ and $x_\mathcal{S} + x_\mathcal{T} < C_\mathcal{S} + C_\mathcal{T}$, and (d) $x_\mathcal{S} < C_\mathcal{S}$ and $x_\mathcal{T} < C_\mathcal{T}$. Using simple algebra it is easy to see that for all of these cases, $J_{\mathcal{S}\cup\mathcal{T}} \leq J_\mathcal{S} + J_\mathcal{T}$ or equivalently,

$$J(x_\mathcal{S} + x_\mathcal{T}, C_\mathcal{S} + C_\mathcal{T}) \leq J(x_\mathcal{S}, C_\mathcal{S}) + J(x_\mathcal{T}, C_\mathcal{T}), \quad (25)$$

and this proves subadditivity of J. Since the storage cost function $u(\mathcal{S}) = J(x_\mathcal{S}, C_\mathcal{S})$, the cost sharing cooperative game $(\mathcal{N}, u)$ is subadditive. □

C. Proof of Theorem 3

We notice that the function J is positive homogeneous, i.e, for any $\alpha \geq 0$, $J(\alpha x_\mathcal{S}, \alpha C_\mathcal{S}) = \alpha J(x_\mathcal{S}, C_\mathcal{S})$. J is also subadditive as per Theorem 2. Thus for any arbitrary balanced map $\alpha : 2^\mathcal{N} \to [0, 1]$

$$\begin{aligned}
&\sum_{\mathcal{S}\in 2^\mathcal{N}} \alpha(\mathcal{S}) u(\mathcal{S}) \\
&\quad = \sum_{\mathcal{S}\in 2^\mathcal{N}} \alpha(\mathcal{S}) J(x_\mathcal{S}, C_\mathcal{S}) \\
&\quad = \sum_{\mathcal{S}\in 2^\mathcal{N}} J(\alpha(\mathcal{S}) x_\mathcal{S}, \alpha(\mathcal{S}) C_\mathcal{S}) \text{ [positive homogeneity]} \\
&\quad \geq J(\sum_{\mathcal{S}\in 2^\mathcal{N}} \alpha(\mathcal{S}) x_\mathcal{S}, \sum_{\mathcal{S}\in 2^\mathcal{N}} \alpha(\mathcal{S}) C_\mathcal{S}) \text{ [subadditivity]} \\
&\quad = J(\sum_{i\in\mathcal{N}} \sum_{\mathcal{S}\in 2^\mathcal{N}} \alpha(\mathcal{S}) \mathbf{1}_\mathcal{S}(i) x_\mathcal{S}, \sum_{i\in\mathcal{N}} \sum_{\mathcal{S}\in 2^\mathcal{N}} \alpha(\mathcal{S}) \mathbf{1}_\mathcal{S}(i) C_\mathcal{S}) \\
&\quad = J(x_\mathcal{N}, C_\mathcal{N}) = u(\mathcal{N}).
\end{aligned}$$

and this proves that the cost sharing game $(\mathcal{N}, u)$ is balanced. □

D. Proof of Theorem 4

We begin by proving that the cost allocation (9) is an imputation, *i.e.* $\xi \in \mathcal{I}$. An imputation is a cost allocation satisfying budget balance and individual rationality.

If $x_\mathcal{N} \geq C_\mathcal{N}$:

$$\sum_{i\in\mathcal{N}} \xi_i = \sum_{i\in\mathcal{N}} \pi_i C_i + \pi_h (x_\mathcal{N} - C_\mathcal{N}) + \pi_\ell C_\mathcal{N} = u(\mathcal{N}).$$

If $x_\mathcal{N} < C_\mathcal{N}$:

$$\sum_{i\in\mathcal{N}} \pi_i C_i + \pi_\ell x_\mathcal{N} = u(\mathcal{N}).$$

Thus, $\sum_{i\in\mathcal{N}} \xi_i = u(\mathcal{N})$ and the cost allocation $\{\xi_i : i \in \mathcal{N}\}$ satisfies budget balance.

The individual cost is:

$$u(\{i\}) = \begin{cases} \pi_i C_i + \pi_h (x_i - C_i) + \pi_\ell C_i & x_i \geq C_i \\ \pi_i C_i + \pi_\ell x_i & x_i < C_i \end{cases}$$

If $x_\mathcal{N} \geq C_\mathcal{N}$:

$$\begin{aligned}
\xi_i &= \pi_i C_i + \pi_h (x_i - C_i) + \pi_\ell C_i \\
&= \pi_i C_i + \pi_\ell x_i - \pi_\delta (C_i - x_i) \\
&= u(\{i\}) - \pi_\delta (C_i - x_i)^+.
\end{aligned}$$

If $x_\mathcal{N} < C_\mathcal{N}$:

$$\begin{aligned}
\xi_i &= \pi_i C_i + \pi_\ell x_i \\
&= u(\{i\}) - \pi_\delta (x_i - C_i)^+.
\end{aligned}$$

Thus, $\xi_i \leq v(\{i\})$ for all $i \in \mathcal{N}$, and the cost allocation ξ is individually rational. Since it is also budget balanced, it is an imputation, *i.e.* $\xi \in \mathcal{I}$.

Finally, to prove that the cost allocation ξ belongs to the core of the cooperative game, we have to prove that $\sum_{i\in\mathcal{S}} \xi_i \leq u(\mathcal{S})$ for any coalition $\mathcal{S} \subseteq \mathcal{N}$.

If $x_\mathcal{N} \geq C_\mathcal{N}$:

$$\begin{aligned}
\sum_{i\in\mathcal{S}} \xi_i &= \sum_{i\in\mathcal{S}} \pi_i C_i + \pi_h (x_\mathcal{S} - C_\mathcal{S}) + \pi_\ell C_\mathcal{S} \\
&= \sum_{i\in\mathcal{S}} \pi_i C_i + \pi_\ell x_\mathcal{S} - \pi_\delta (C_\mathcal{S} - x_\mathcal{S}) \\
&= u(\mathcal{S}) - \pi_\delta (C_\mathcal{S} - x_\mathcal{S})^+.
\end{aligned}$$

[1] Since $x_\mathcal{S}$, $x_\mathcal{T}$, $C_\mathcal{S}$ and, $C_\mathcal{T}$ are arbitrary nonnegative real numbers, any other possible case can be easily recast as one of these four cases by interchanging $\mathcal{S}$ and $\mathcal{T}$.

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If $x_{\mathcal{N}} < C_{\mathcal{N}}$:

$$\begin{aligned}\sum_{i\in\mathcal{S}} \xi_i &= \sum_{i\in\mathcal{S}} \pi_{\mathcal{S}} C_{\mathcal{S}} + \pi_\ell x_{\mathcal{S}} \\ &= u(\mathcal{S}) - \pi_\delta (x_{\mathcal{S}} - C_{\mathcal{S}})^+.\end{aligned}$$

Thus, $\sum_{i\in\mathcal{S}} \xi_i \le u(\mathcal{S})$ for any $\mathcal{S} \subseteq \mathcal{N}$ and the cost allocation ξ is in the core of the cooperative game $(\mathcal{N}, u)$. □

E. Proof of Theorem 6

Let $\mathcal{S}$ and $\mathcal{T}$ two arbitrary nonempty disjoint coalitions, *i.e.* $\mathcal{S}, \mathcal{T} \subseteq \mathcal{N}$ such that $\mathcal{S} \cap \mathcal{T} = \emptyset$. Define

$$\Phi(\mathbf{x}_{\mathcal{S}}) = \min_{C_{\mathcal{S}} \ge 0} \mathbb{E} J(C_{\mathcal{S}}, \mathbf{x}_{\mathcal{S}}). \tag{26}$$

We shall prove that $\Phi(\mathbf{x}_{\mathcal{S}})$ is a subbadditive function.

From the definition of J given in (4),

$$J(\mathbf{x}_{\mathcal{S}}, C^*_{\mathcal{S}}) + J(\mathbf{x}_{\mathcal{T}}, C^*_{\mathcal{T}}) \ge J(\mathbf{x}_{\mathcal{S}} + \mathbf{x}_{\mathcal{T}}, C^*_{\mathcal{S}} + C^*_{\mathcal{T}}).$$

Taking expectations on both sides,

$$\begin{aligned}\Phi(\mathbf{x}_{\mathcal{S}}) + \Phi(\mathbf{x}_{\mathcal{T}}) &\ge \mathbb{E} J(\mathbf{x}_{\mathcal{S}} + \mathbf{x}_{\mathcal{T}}, C^*_{\mathcal{S}} + C^*_{\mathcal{T}}) \\ &\ge \min_{C\ge 0} \mathbb{E} J(\mathbf{x}_{\mathcal{S}} + \mathbf{x}_{\mathcal{T}}, C) \\ &= \Phi(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{T}}),\end{aligned}$$

and this proves subadditivity of Φ.

Subadditivity of the cost sharing cooperative game $(\mathcal{N}, v)$ is a consequence of the subadditivity of Φ because $v(\mathcal{S}) = \Phi(\mathbf{x}_{\mathcal{S}})$ for any $\mathcal{S} \subseteq \mathcal{N}$. □

F. Proof of Theorem 7

First, we prove that the function Φ defined by (26) is positive homogeneous. Observe that if a random variable z has CDF F, then the scaled random variable αz with $\alpha > 0$ has CDF: $F_\alpha(\theta) = \mathbb{P}\{\alpha z \le \theta\} = F(\theta/\alpha)$. Then, for any $\alpha \ge 0$ and $\gamma \in [0,1]$, $\gamma = F(C)$ if and only if $\gamma = F_\alpha(\alpha C)$. This means that if $C_{\mathcal{S}}$ is such that $\Phi(\mathbf{x}_{\mathcal{S}}) = \mathbb{E}J(\mathbf{x}_{\mathcal{S}}, C^*_{\mathcal{S}})$, then $\Phi(\alpha \mathbf{x}_{\mathcal{S}}) = \mathbb{E}J(\alpha \mathbf{x}_{\mathcal{S}}, \alpha C^*_{\mathcal{S}})$.

For any $\alpha \ge 0$, and from the definition of the daily storage cost J (4), $J(\alpha \mathbf{x}_{\mathcal{S}}, \alpha C^*_{\mathcal{S}}) = \alpha J(\mathbf{x}_{\mathcal{S}}, C^*_{\mathcal{S}})$. Taking expectations on both sides, $\Phi(\alpha \mathbf{x}_{\mathcal{S}}) = \alpha \Phi(\mathbf{x}_{\mathcal{S}})$, and this proves positive homogeneity of Φ.

Now, balancedness of the cost sharing cooperative game $(\mathcal{N}, v)$ is a consequence of the properties of function Φ

$$\begin{aligned}\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S}) v(\mathcal{S}) &= \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S}) \Phi(\mathbf{x}_{\mathcal{S}}) \\ &= \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \Phi(\alpha(\mathcal{S}) \mathbf{x}_{\mathcal{S}}) \text{ [positive homogeneity]} \\ &\ge \Phi(\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S}) \mathbf{x}_{\mathcal{S}}) \text{ [subadditivity]} \\ &= \Phi(\sum_{i\in\mathcal{N}} \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S}) \mathbf{1}_{\mathcal{S}}(i) \mathbf{x}_{\mathcal{S}}) \\ &= \Phi(\mathbf{x}_{\mathcal{N}}) = v(\mathcal{N}).\end{aligned}$$

G. Proof of Theorem 8

We begin by proving that the cost allocation given by (9) satisfies budget balance,

$$\begin{aligned}\sum_{i\in\mathcal{N}} \zeta_i &= \sum_{i\in\mathcal{N}} \pi_\ell \mathbb{E}[\mathbf{x}_i] + \sum_{i\in\mathcal{N}} \pi_{\mathcal{S}} \mathbb{E}[\mathbf{x}_i \mid \mathbf{x}_{\mathcal{N}} \ge C^*_{\mathcal{N}}] \\ &= \pi_\ell \mathbb{E}\left[\sum_{i\in\mathcal{N}} \mathbf{x}_i\right] + \pi_{\mathcal{S}} \mathbb{E}\left[\sum_{i\in\mathcal{N}} \mathbf{x}_i \mid \mathbf{x}_{\mathcal{N}} \ge C^*_{\mathcal{N}}\right] \\ &= \pi_\ell \mathbb{E}[\mathbf{x}_{\mathcal{N}}] + \pi_h \mathbb{E}[\mathbf{x}_{\mathcal{N}} \mid \mathbf{x}_{\mathcal{N}} \ge C^*_{\mathcal{N}}] \\ &= v(\mathcal{N}).\end{aligned}$$

The cost allocation is in the core if we prove that $v(\mathcal{S}) \ge \sum_{i\in\mathcal{S}} \zeta_i$ for any coalition $\mathcal{S} \subset \mathcal{N}$. Please note that individual rationality is included in the previous condition.

The storage cost for a coalition $\mathcal{S} \subset \mathcal{N}$ is

$$\begin{aligned}v(\mathcal{S}) &= \pi_\ell \mathbb{E}[\mathbf{x}_{\mathcal{S}}] + \pi_{\mathcal{S}} \mathbb{E}[\mathbf{x}_{\mathcal{S}} \mid \mathbf{x}_{\mathcal{S}} \ge C^*_{\mathcal{S}}] \\ &= \pi_{\mathcal{S}} C^*_{\mathcal{S}} + \pi_h \mathbb{E}[(\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}})^+] + \pi_\ell \mathbb{E}[\min\{C^*_{\mathcal{S}}, \mathbf{x}_{\mathcal{S}}\}].\end{aligned}$$

Note that

$$\pi_h (\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}})^+ + \pi_\ell \min\{C^*_{\mathcal{S}}, \mathbf{x}_{\mathcal{S}}\} \ge \pi_h (\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}}) + \pi_\ell C^*_{\mathcal{S}}.$$

and therefore,

$$\begin{aligned}&\pi_{\mathcal{S}} C^*_{\mathcal{S}} + \pi_h \mathbb{E}[(\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}})^+] + \pi_\ell \mathbb{E}[\min\{C^*_{\mathcal{S}}, \mathbf{x}_{\mathcal{S}}\}] \\ &\qquad \ge \pi_{\mathcal{S}} C^*_{\mathcal{S}} + \pi_h \mathbb{E}[(\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}})] + \pi_\ell C^*_{\mathcal{S}}.\end{aligned}$$

Let us define the sets $\mathcal{A}_+ = \{\mathbf{x}_{\mathcal{N}} \in \mathbb{R}_+ \mid \mathbf{x}_{\mathcal{N}} \ge C_{\mathcal{N}}\}$, $\mathcal{A}_- = \mathbb{R}_+ \backslash \mathcal{A}_+$, and the auxiliary function $\psi(\mathbf{x}_{\mathcal{N}})$ as follows

$$\psi(\mathbf{x}_{\mathcal{N}}) = \begin{cases} \pi_h & \text{if } \mathbf{x}_{\mathcal{N}} \in \mathcal{A}_+ \\ \pi_\ell & \text{if } \mathbf{x}_{\mathcal{N}} \in \mathcal{A}_- \end{cases}$$

Let $F(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}})$ be the joint distribution function of the peak consumptions $(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}})$, then

$$\begin{aligned}&\mathbb{E}[\psi(\mathbf{x}_{\mathcal{N}})(\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}})] \\ &= \pi_\ell \int_{\mathbb{R}_+} \int_{\mathcal{A}_-} (\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}}) dF(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}}) + \\ &\qquad \pi_h \int_{\mathbb{R}_+} \int_{\mathcal{A}_+} (\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}}) dF(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}}) \\ &\le \pi_h \int_{\mathbb{R}_+} \int_{\mathcal{A}_+ \cup \mathcal{A}_-} (\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}}) dF(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}}) \\ &= \pi_h \int_{\mathbb{R}_+} (\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}}) dF(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}}) \\ &= \mathbb{E}[(\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}}],\end{aligned}$$

and consequently,

$$\begin{aligned}&\pi_{\mathcal{S}} C^*_{\mathcal{S}} + \pi_h \mathbb{E}[(\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}})] + \pi_\ell C^*_{\mathcal{S}} \\ &\qquad \ge \pi_{\mathcal{S}} C^*_{\mathcal{S}} + \mathbb{E}[\psi(\mathbf{x}_{\mathcal{N}})(\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}})] + \pi_\ell C^*_{\mathcal{S}}\end{aligned}$$

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Now, we prove that the right hand side of the previous expression equals $\sum_{i\in\mathcal{S}} \zeta_i$

$$\begin{aligned}
&\pi_{\mathcal{S}} C^*_{\mathcal{S}} + \mathbb{E}[\psi(\mathbf{x}_{\mathcal{N}})(\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}})] + \pi_\ell C^*_{\mathcal{S}} \\
&= \pi_{\mathcal{S}} C^*_{\mathcal{S}} + \int_{R_+}\int_{R_+} \psi(\mathbf{x}_{\mathcal{N}})(\mathbf{x}_{\mathcal{N}} - C^*_{\mathcal{N}}) dF(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}}) + \pi_\ell C^*_{\mathcal{S}} \\
&= \pi_{\mathcal{S}} C^*_{\mathcal{S}} + \pi_\ell \int_{\mathbb{R}_+}\int_{\mathcal{A}_- \cup \mathcal{A}_+} (\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}}) dF(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}}) + \\
&\qquad (\pi_h - \pi_\ell) \int_{\mathbb{R}_+}\int_{\mathcal{A}_+} (\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}}) dF(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}}) + \pi_\ell C^*_{\mathcal{S}} \\
&= \pi_{\mathcal{S}} C^*_{\mathcal{S}} + \pi_\delta \int_{\mathbb{R}_+}\int_{\mathcal{A}_+} (\mathbf{x}_{\mathcal{S}} - C^i_{\mathcal{S}}) dF(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}}) + \pi_\ell \mathbb{E}[\mathbf{x}_i] \\
&= \pi_{\mathcal{S}} C^*_{\mathcal{S}} + \frac{\pi_{\mathcal{S}}}{1-\gamma_{\mathcal{S}}} \int_{\mathbb{R}_+}\int_{\mathcal{A}_+} (\mathbf{x}_{\mathcal{S}} - C^*_{\mathcal{S}}) dF(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}}) + \pi_\ell \mathbb{E}[\mathbf{x}_i] \\
&= \pi_{\mathcal{S}} \frac{1}{\mathbb{P}\{\mathbf{x}_{\mathcal{N}} \geq C_{\mathcal{N}}\}} \int_{\mathbb{R}_+}\int_{\mathcal{A}_+} \mathbf{x}_{\mathcal{S}} dF(\mathbf{x}_{\mathcal{S}}, \mathbf{x}_{\mathcal{N}}) + \pi_\ell \mathbb{E}[\mathbf{x}_i] \\
&= \sum_{i\in\mathcal{S}} \zeta_i
\end{aligned}$$

Thus, $\sum_{i\in\mathcal{S}} \zeta_i \leq v(\mathcal{S})$ and the cost allocation $\{\zeta_i : i \in \mathcal{N}\}$ is an imputation in the core.

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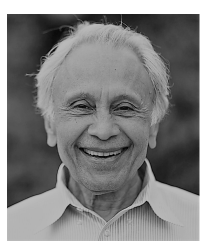

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