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A strategy to maintain short-term stability and long-term profitability of renewable energy aggregation

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***Abstract*— In this paper, we develop a strategy to enhance cooperation among a group of renewable energy producers (REP). The REPs form a coalition that is managed by an aggregator. In the coalition, the REPs desire an allocation of the realized profit at each time that keeps them in the coalition (*i.e.*, a stabilizing allocation). We first compute a necessary and sufficient condition of developing the joint contract under this scenario. Though the REPs get a stable allocation under our scheme, they lose some overall profit in the long-term. So next, we develop a strategy that can be used by the aggregator to improve the long-term profit of this coalition. The proposed method will support coalition formation and thus promote market integration of renewable energy. We also develop a numerical example to illustrate our results.**

I. INTRODUCTION

Renewable power produced by wind and solar power plants is uncertain (the power generation can not be known in advance), intermittent (the output fluctuates with time), and non-dispatchable (the output can not be controlled). These three characteristics are captured by the term variability [1]. Variability is a significant challenge associated with the large scale renewable integration [2]

Aggregation of geographically diverse wind and solar energy power plants reduces their variability [3]. This reduction in variability is due to low correlation that may exist among them. We consider a set of REPs interested in forming a coalition, implemented through an aggregator, to take advantage of this potential reduction in variability. From an economic viewpoint, if reduction in variability can be used to generate additional profits, then their fair distribution is critical to keeping the coalition stable. Thus, we consider a scenario where the aggregator of the coalition bids in the day-ahead market of a two settlement market set-up. In a previous paper [4], the optimal contract to maximize the expected profit was derived. Moreover, a balanced coalition game was formulated where an allocation in the core were obtained. However, the realized profit for any given period of time can be very different from the expected profit, and the mechanisms developed to allocate the expected profit may not be satisfactory for the realized profit. Some strategies have been proposed in the literature to deal with this issue. In [5], [6], an aggregator allocates the realized profit. But none of the proposed allocations are in the core and there is no way to ensure stable coalition. Consequently, they have modeled the interaction among the REPs using non-cooperative games and shown existence of Nash equilibrium. In [7], the authors used the allocation of [5] and modified it by a factor in order to improve the average profit of the agents. In our previous work [8], we proposed to use a separable contract that produces a balanced coalitional game and we developed a profit allocation in the core such that the REPs achieve a stable sharing of the realized profit at each time. However, the coalition of REPs still lose profit in the long-term, because they bid a stabilizing contract that does not maximize the expected profit. In this paper, we are concerned with the problem of recovering this profit while maintaining the stability of the coalition.

Our main contribution is a two step strategy that ensures the formation of stable coalitions in the short-term and results in recovering additional profitability in the long-term. Our approach is structured around the following building blocks: (i) a coalition managed by an aggregator, (ii) a separable joint contract of the group which we prove is the only contract that allows to share the realized profit in a stabilized way, and (iii) two cost causation based allocations at different time scales. The short-term allocation makes use of the separable joint contract that stabilizes the coalitional game of the realized profit. But the REPs will lose some profit in the long-term if this contract is bid in the market. In our strategy, to recover this lost opportunity, the aggregator bids a joint contract that maximizes the expected profit of the group but shares the realized profit using the stabilizing joint contract. The long-term allocation uses the difference between the stabilizing contract and the contract maximizing the expected profit to share the recovered profit after a sufficiently large interval of time to ensure that the additional profit is positive. Both profit allocations rely on the cost causation principles [9], [10]. Since the additional profit can be negative for certain time periods, the aggregator needs cash to operate. In order to successfully implement this two step strategy, the aggregator needs to compute two key parameters: the length of the long-term time interval and the required cash to operate. A methodology to compute these parameters using statistical inference is proposed. A numerical example has been developed to show that the proposed strategy can be successfully implemented.

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The remainder of the paper is organized as follows. In Section II, we describe our previous results and formulate the problem. In Section III, we derive the necessary and sufficient condition for a joint stabilizing contract and also develop an allocation of the deviation cost. In Section IV, the strategy to improve the long-term profit of these REPs is developed. A numerical example is presented in Section V. Finally we present the conclusions of the paper in Section VI.

II. Problem formulation with previous results

Consider a set of N independent renewable energy producers (REPs) indexed by $i \in \mathcal{N} := \{1, 2, \dots, N\}$. We consider a two-settlement market system consisting of an *ex-ante* forward market and an *ex-post* imbalance mechanism to penalize uninstructed contract deviations. Power producers bid contracts for time intervals of length T. We concentrate here on the k-th interval denoted by $\Omega_k := [(k-1)T, kT]$. The price in *ex-ante* forward market for the interval Ω_k is denoted by p_k and is assumed to be constant and known for every REP[1]. The penalty prices are modeled by random variables (q, λ), where q is the shortfall penalty price and λ is the surplus penalty price. Their realized values for the interval Ω_k are q_k and λ_k, respectively.

REPs may form coalitions to aggregate their renewable power production and jointly bid into electricity markets. The power produced by each REP i at time t is modeled by a random variable $w_i(t)$ with support $[0, W_i]$, where W_i is the nameplate production capacity. The power produced by a coalition $\mathcal{S} \subseteq \mathcal{N}$ of REPs is given by the random variable $w_{\mathcal{S}}(t) = \sum_{i \in \mathcal{S}} w_i(t)$ with support $[0, W^{\mathcal{S}}]$, where $W^{\mathcal{S}}$ is the nameplate capacity of the subset $\mathcal{S}$, *i.e.* $W^{\mathcal{S}} = \sum_{i \in \mathcal{S}} W^i$. The cumulative probability distribution function of the aggregated power is $F_{\mathcal{S}}(x, t) = P(w_{\mathcal{S}}(t) \le x)$. Let the joint contractual power bid of the coalition be $C_{\mathcal{S}}(\Omega_k)$, which is constant during the time interval Ω_k. In this model, the realized profit of a coalition for the time interval Ω_k is

$$\Pi(w_{\mathcal{S}}(\Omega_k), C_{\mathcal{S}}(\Omega_k)) = \int_{\Omega_k} \pi_t^k(w_{\mathcal{S}}(t), C_{\mathcal{S}}(\Omega_k)) dt \quad (1)$$

where

$$\pi_t^k(w_{\mathcal{S}}(t), C_{\mathcal{S}}(\Omega_k)) = p_k C_{\mathcal{S}}(\Omega_k) - q_k (C_{\mathcal{S}}(\Omega_k) - w_{\mathcal{S}}(t))_+ - \lambda_k (w_{\mathcal{S}}(t) - C_{\mathcal{S}}(\Omega_k))_+, \quad (2)$$

and $(x)_+ := \max\{0, x\}$. We also define the deviation of realized value from the contract as $d_{\mathcal{S}}(t) = w_{\mathcal{S}}(t) - C_{\mathcal{S}}(\Omega_k)$ and the cost related to deviation as

$$\theta^k(d_{\mathcal{S}}(t), q_k, \lambda_k) = \lambda_k (d_{\mathcal{S}}(t))_+ + q_k(-d_{\mathcal{S}}(t))_+. \quad (3)$$

The expected profit and the expected profit maximizing contract are defined as

$$J_k(\mathcal{S}) = \mathbb{E}\Pi(w_{\mathcal{S}}(\Omega_k), C_{\mathcal{S}}(\Omega_k)), \quad (4)$$

$$C^*_{\mathcal{S}}(\Omega_k) = \arg\max_{C \ge 0} J_k(\mathcal{S}). \quad (5)$$

[1]It means that either all the REPs are in the same bus or the network is not congested.

The result on optimal contract for maximizing expected profit was obtained in [4].

Theorem 1: The optimal contract is given by

$$C^*_{\mathcal{S}}(\Omega_k) = F^{-1}_{\mathcal{S}}(\gamma_k; \Omega_k) \quad (6)$$

where

$$\gamma_k = \frac{p_k + \mu_\lambda}{\mu_q + \mu_\lambda} \quad (7)$$

and $\mu_q = \mathbb{E}[q]$, $\mu_\lambda = \mathbb{E}[\lambda]$.

The optimal expected profit is given by

$$\frac{J_k(\mathcal{S})}{T} = \mu_q \int_0^\gamma F^{-1}_{\mathcal{S}}(x; \Omega_k) dx - \mu_\lambda \int_\gamma^1 F^{-1}_{\mathcal{S}}(x; \Omega_k) dx$$

The previous theorem is also applicable to an individual REP by choosing $\mathcal{S} = \{i\}$.

A cooperative game theoretic model [11] of wind power producers is formed where the optimal contract of any coalition $\mathcal{S}$ is given by $C^*_{\mathcal{S}}(\Omega_k)$. A cooperative pay-off maximization game is effective when the game can be proved to be supperadditive and balanced. Superadditivity implies profitability of the agents by joining larger coalitions. Balancedness implies the existence of stabilizing or stable profit allocations. Under a stabilizing profit allocation, the profit of the grand coalition can be suitably allocated among the members in such a way that no player makes an extra profit by leaving the grand coalition and making separate smaller coalitions. The set containing all the stabilizing profit allocations is the *core* of the game. More details about cooperative game theory can be found in [11].

It has been proved that the cooperative game of expected profit with the contract $C^*_{\mathcal{S}}(\Omega_k)$ has a non empty core [4]. As a sharing rule, a particular imputation is nucleolus. Another imputation that minimizes the worst-case excess was proposed since it is computationally less demanding. But the cooperative game of realized profit using this optimal contract has an empty core [8]. The major implication of this fact is that it is not possible to have a satisfactory allocation of realized profit with the joint contract that maximizes the expected profit and the coalition is not sustainable. In [8], it has been shown that the joint contract $C_{\mathcal{S}}(\Omega_k) = \sum_{i \in \mathcal{S}} C_i(\Omega_k)$, where $C_i(\Omega_k)$ is any contract for the i-th REP, is such that core of the cooperative game of realized profit is non-empty and an allocation in the core with analytical formula was derived. But what is the condition on the joint contract such that the core of the cooperative game of realized profit is always non-empty? In the next section, we will derive a necessary and sufficient condition on the joint contract such that the core of the realized profit game is non-empty and thus the cooperative game have a stable allocation of the realized profit.

III. Necessary and Sufficient Condition on the Joint Contract for Stabilized Allocation of the Realized Profit

Here we focus on the cooperative game for the case of realized profit. Let $\mathcal{N}$ be the set of REPs and $v : 2^{\mathcal{N}} \to \mathbb{R}$

denote the payoff function defined as follows:

$$v(\mathcal{S}) = \Pi(w_{\mathcal{S}}(\Omega_k), C_{\mathcal{S}}(\Omega_k)),\ \mathcal{S} \in \mathcal{N}. \tag{8}$$

We begin by investigating the condition on $C_{\mathcal{S}}(\Omega_k)$ that make the coalitional game $(\mathcal{N}, v)$ supperadditive and balanced for an interval Ω_k. The following theorem establishes a necessary and sufficient condition for superadditivity.

Theorem 2: The coalitional game $(\mathcal{N}, v)$ is supperadditive if and only if the joint contract is additive, *i.e.* $C_{\mathcal{N}}(\Omega_k) = \sum_{i\in\mathcal{N}} C_i(\Omega_k)$.

Proof.: Let us consider two disjoint coalitions $\mathcal{S}$ and $\mathcal{T}$ and let $\mathcal{S} \cup \mathcal{T} = \mathcal{P}$. For any $t \in \Omega_k$, It holds that

$$w_{\mathcal{S}}(t) + w_{\mathcal{T}}(t) = w_{\mathcal{P}}(t). \tag{9}$$

For superadditivity we need to prove,

$$v(\mathcal{S}) + v(\mathcal{T}) \leq v(\mathcal{P}), \tag{10}$$

or equivalently

$$\begin{aligned} \Pi(w_{\mathcal{S}}(\Omega_k), C_{\mathcal{S}}(\Omega_k)) &+ \Pi(w_{\mathcal{T}}(\Omega_k), C_{\mathcal{T}}(\Omega_k)) \\ &\leq \Pi(w_{\mathcal{P}}(\Omega_k), C_{\mathcal{P}}(\Omega_k)). \end{aligned} \tag{11}$$

Consider a case when realized power produced by REPs are such that $w_{\mathcal{S}}(t) \leq C_{\mathcal{S}}(\Omega_k)$, $w_{\mathcal{T}}(t) \leq C_{\mathcal{T}}(\Omega_k)$, $w_{\mathcal{P}}(t) \leq C_{\mathcal{P}}(\Omega_k)$, for all $t \in \Omega_k$ with $\mathcal{S}, \mathcal{T}, \mathcal{P} \subseteq \mathcal{N}$. Thus, upon simplification we need to prove

$$(p_k - q_k)(C_{\mathcal{S}}(\Omega_k) + C_{\mathcal{T}}(\Omega_k)) \leq (p_k - q_k)C_{\mathcal{P}}(\Omega_k). \tag{12}$$

Since the previous inequality must hold for any $p_k - q_k$, this is only possible if

$$C_{\mathcal{S}}(\Omega_k) + C_{\mathcal{T}}(\Omega_k) = C_{\mathcal{P}}(\Omega_k). \tag{13}$$

Considering all possible $\mathcal{S}$ and $\mathcal{T}$, $C_{\mathcal{N}}(\Omega_k) = \sum_{i\in\mathcal{N}} C_i(\Omega_k)$ and this condition is a necessary condition for superadditivity. To prove that it is also a sufficient condition, we only have to apply the properties of the positive part function $(\cdot)_+$. Using (9) and (13)

$$\begin{aligned} (w_{\mathcal{S}}(t) - C_{\mathcal{S}}(\Omega_k)) &+ (w_{\mathcal{T}}(t) - C_{\mathcal{T}}(\Omega_k)) \\ &= (w_{\mathcal{P}} - C_{\mathcal{P}}(\Omega_k)) \end{aligned}$$

implies

$$\begin{aligned} (w_{\mathcal{S}}(t) - C_{\mathcal{S}}(\Omega_k))_+ &+ (w_{\mathcal{T}}(t) - C_{\mathcal{T}}(\Omega_k))_+ \\ &\geq (w_{\mathcal{P}}(t) - C_{\mathcal{P}}(\Omega_k))_+, \end{aligned} \tag{14}$$

and

$$\begin{aligned} (C_{\mathcal{S}}(\Omega_k) - w_{\mathcal{S}}(t)) &+ (C_{\mathcal{T}}(\Omega_k) - w_{\mathcal{T}}(t)) \\ &= (C_{\mathcal{P}}(\Omega_k) - w_{\mathcal{P}}(t)) \end{aligned}$$

implies

$$\begin{aligned} (C_{\mathcal{S}}(\Omega_k) - w_{\mathcal{S}}(t))_+ &+ (C_{\mathcal{T}}(\Omega_k) - w_{\mathcal{T}}(t))_+ \\ &\geq (C_{\mathcal{P}}(\Omega_k) - w_{\mathcal{P}}(t))_+. \end{aligned} \tag{15}$$

Now, multiplying (13) by p_k, (14) by $-q_k$, (15) by $-\lambda_k$ and adding them gives

$$\begin{aligned} \pi_t^k(w_{\mathcal{S}}(t), C_{\mathcal{S}}(\Omega_k)) &+ \pi_t^k(w_{\mathcal{T}}(t), C_{\mathcal{T}}(\Omega_k)) \\ &\leq \pi_t^k(w_{\mathcal{P}}(t), C_{\mathcal{P}}(\Omega_k)). \end{aligned} \tag{16}$$

Integrating (16) for the time interval Ω_k, we get

$$\begin{aligned} \Pi(w_{\mathcal{S}}(\Omega_k), C_{\mathcal{S}}(\Omega_k)) &+ \Pi(w_{\mathcal{T}}(\Omega_k), C_{\mathcal{T}}(\Omega_k)) \\ &\leq \Pi(w_{\mathcal{P}}(\Omega_k), C_{\mathcal{P}}(\Omega_k)). \end{aligned} \tag{17}$$

Thus, the coalitional game is supperadditive if and only if $C_{\mathcal{N}}(\Omega_k) = \sum_{i\in\mathcal{N}} C_i(\Omega_k)$. □

Next, we prove balancedness of the game when the coalition contract is additive, *i.e.* $C_{\mathcal{N}}(\Omega_k) = \sum_{i\in\mathcal{N}} C_i(\Omega_k)$. The result is summarized in the following theorem.

Theorem 3 (Balancedness): The coalitional game $(\mathcal{N}, v)$ with $C_{\mathcal{N}}(\Omega_k) = \sum_{i\in\mathcal{N}} C_i(\Omega_k)$ is balanced.

Proof: The payoff function of the game v is homogeneous, because for any $\mathcal{S} \subseteq N$,

$$\begin{aligned} \lambda(\mathcal{S}) \int_{\Omega_k} \pi_t(w_{\mathcal{S}}(t), C_{\mathcal{S}}(\Omega_k))dt &= \\ \int_{\Omega_k} \pi_t(\lambda(\mathcal{S})w_{\mathcal{S}}(t), \lambda(S)C_{\mathcal{S}}(\Omega_k))dt& \end{aligned} \tag{18}$$

The payoff function is also superadditive as per Theorem 2. Then, for any arbitrary balanced map $\alpha : 2^{\mathcal{N}} \rightarrow [0, 1]$, *i.e.* a map satisfying $\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})\mathbf{1}_{\mathcal{S}}(i) = 1$, where $\mathbf{1}_{\mathcal{S}}$ denotes the indicator function of the set $\mathcal{S}$,

$$\begin{aligned} \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})v(\mathcal{S}) &= \sum_{\mathcal{S}\in 2^{\mathcal{N}}} v(\alpha(\mathcal{S})\mathcal{S}) \quad (v \text{ is homogeneous}) \\ &\leq v\left(\sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})\mathcal{S}\right) (v \text{ is supperadditive}) \\ &= v\left(\sum_{i\in\mathcal{N}} \sum_{\mathcal{S}\in 2^{\mathcal{N}}} \alpha(\mathcal{S})\mathbf{1}_{\mathcal{S}}(i)\right) = v(\mathcal{N}). \end{aligned}$$

And this proves that the game is balanced and the core is non empty. □

We have earlier shown that $C_{\mathcal{N}}(\Omega_k) = \sum_{i=1}^{N} C_i(\Omega_k)$ allows to allocate realized profit in a stable way [8]. In the theorems stated above, we have proved that it is in fact the only contract that makes realized profit allocation stable.

We conclude this section by providing a stabilizing allocation for the coalitional game of the realized profit with the additive contract $C_{\mathcal{N}}(\Omega_k) = \sum_{i=1}^{N} C_i(\Omega_k)$. A profit allocation $\{\xi_i^k, i \in \mathcal{N}\}$ for the interval Ω_k is a distribution of the profit Π_k among the game participants. The profit allocation is satisfactory for all the REPs if it is in the core of the coalitional game [11]. Consider the following allocation.

Allocation 1 (Short-Term Profit Allocation): An allocation of the realized profit for the grand coalition $\mathcal{N}$ that bids the additive contract $C_{\mathcal{N}}(\Omega_k) = \sum_{i\in\mathcal{N}} C_i(\Omega_k)$ for the short-term time interval Ω_k in the two settlement market is given by:

$$\xi_i^k = \int_{\Omega_k} \xi_i^k(t)dt, \tag{19}$$

where

$$\xi_i^k(t) = \begin{cases} p_k C_i(\Omega_k), & \text{if } d_{\mathcal{N}}(t) = 0, \\ p_k C_i(\Omega_k) + q_k d_i(t), & \text{if } d_{\mathcal{N}}(t) > 0, \\ p_k C_i(\Omega_k) - \lambda_k d_i(t), & \text{if } d_{\mathcal{N}}(t) < 0. \end{cases} \tag{20}$$

The following theorem establishes that Allocation 1 of the realized profit with additive contract is in the core of the game.

Theorem 4 ([8]): The profit allocation given by Allocation 1 is in the core of the cooperative game.

Since the joint contract is separable among the REPs, only the deviation cost $\theta(d_S(t), q_k, \lambda_k)$ is allocated as follows.

$$\gamma_i^k(t) = \begin{cases} 0, & \text{if } d_{\mathcal{N}}(t) = 0, \\ -q_k d_i(t), & \text{if } d_{\mathcal{N}}(t) > 0, \\ +\lambda_k d_i(t), & \text{if } d_{\mathcal{N}}(t) < 0. \end{cases} \tag{21}$$

This allocation of the instantaneous deviation cost is shown to be based on cost causation principle in [9], while the corresponding allocation of the instantaneous realized profit $\{\xi_i^k : i \in \mathcal{N}\}$ is shown to follow standalone cost principle (*i.e.* it belongs to the core of the game) in [8].

IV. A Causation Based Allocation of Long-Term Extra Profit

The joint contract must be separable in individual contracts to obtain a satisfactory profit allocation of the realized profit, but each REP has the freedom to choose her contract. If the REP is rational and does not join any coalition, she will choose C_i^*, the contract that optimizes her individual expected profit. Here, we allow the REP to choose her contract according to her individual priorities and denote it by $C_i^+(\Omega_k)$. So the combined bid will be the sum of individual bids, *i.e.* $C_{\mathcal{S}}^+(\Omega_k) = \sum_{i\in\mathcal{S}} C_i^+(\Omega_k)$. However, the combined bid does not necessarily maximize the expected profit of the group, and the REPs will lose some long-term revenue. We consider an aggregator that manages the grand coalition to be a welfare maximizer. The aggregator wants to assign maximum benefit to all the agents of the grand coalition. So we propose a strategy designed by the aggregator to recover the loss in long-term revenue.

A. Strategy to Improve Long-Term Profit

The aggregator will bid the contract that maximizes the expected profit of the group of REPs $C_{\mathcal{S}}^*$ to the market, but it will communicate the REPs that it is bidding $C_{\mathcal{S}}^+$. The difference in realized profit for each interval Ω_k is

$$\Delta\Pi_k = \Pi(w_{\mathcal{S}}(\Omega_k), C_{\mathcal{S}}^*(\Omega_k)) - \Pi(w_{\mathcal{S}}(\Omega_k), C_{\mathcal{S}}^+(\Omega_k)). \tag{22}$$

This quantity can be positive or negative. However, assuming that the renewable power production is cyclostationary [12], we can take a large number of subintervals $\{\Omega_k : k = 1, \dots, n\}$, each with length T and the total difference in profit will be positive. There exists empirical evidence that wind velocity and solar radiation can be modeled as doubly cyclostationary random processes [13], [14] with daily and yearly cycles. By choosing $n = 24\ell$, we can consider that the extra profit that is accumulated to the aggregator is

$$\Delta\Pi = \sum_{k=1}^{n} \Delta\Pi_k \tag{23}$$

and can be decomposed as a sum of stationary random variables

$$\Delta\Pi = \sum_{\ell=1}^{n/24} \left(\sum_{k=24(\ell-1)}^{24\ell-1} \Delta\Pi_k \right), \tag{24}$$

whenever the days ℓ are in the same seasonal period.

The aggregator will distribute the extra profit to all the REPs in a satisfactory way after the time nT is over as a gift of cooperation. This strategy will promote integration of renewables. In order to implement this strategy, the aggregator needs to have some cash initially, as the extra profit can be negative in some intervals. But the aggregator knows that it will have extra profit after a long term that it can distribute to the agents.

B. Contribution of the REPs to the extra profit

In this subsection, we analyze the contribution of the REPs on the extra profit. This will allow us to design a cost causation based allocation of this profit. Let us define the contract deviation $\Delta C_{\mathcal{N}}^+$ as follows:

$$\Delta C_{\mathcal{N}}^+(\Omega_k) = C_{\mathcal{N}}^*(\Omega_k) - C_{\mathcal{N}}^+(\Omega_k)$$

Theorem 5: Let $\sum_{k=1}^n |\Delta C_{\mathcal{N}}^+(\Omega_k)| = 0$, then there is no extra profit for the interval $\cup_{k=1}^n \Omega_k$, *i.e.* $\Delta\Pi = \sum_{k=1}^n \Delta\Pi_k = 0$

Proof: $\sum_{k=1}^n |\Delta C_{\mathcal{N}}^+(\Omega_k)| = 0$ is equivalent to $\Delta C_{\mathcal{N}}^*(\Omega_k) = 0$ for $k = 1, \dots, n$, thus by using (22) and (23), the extra profit is $\Pi_k = 0$ for $k = 1, \dots, n$ and $\sum_{k=1}^n \Pi_k = 0$. ■

As a consequence of Theorem 5, we consider $\sum_{k=1}^n |\Delta C_{\mathcal{N}}^*(\Omega_k)|$ to be the cause of extra profit for the interval $\cup_{k=1}^n \Omega_k$.

The aggregated deviation with respect to the contract that maximizes the expected profit of the grand coalition $\mathcal{N}$ for the time interval Ω_k can be decomposed as follows:

$$\begin{aligned} d_{\mathcal{N}}^*(t) &= w_{\mathcal{N}}(t) - C_{\mathcal{N}}^*(\Omega_k) \\ &= \sum_{i\in\mathcal{N}} (w_i(t) - C_i^+(\Omega_k)) + \sum_{i\in\mathcal{N}} C_i^+(\Omega_k) - C_{\mathcal{N}}^*(\Omega_k) \\ &= \sum_{i\in\mathcal{N}} d_i^+(t) - (C_{\mathcal{N}}^*(\Omega_k) - C_{\mathcal{N}}^+(\Omega_k)) \\ &= d_{\mathcal{N}}^+(t) - \Delta C_{\mathcal{N}}^+(\Omega_k), \end{aligned} \tag{25}$$

then

$$\begin{aligned} |\Delta C_{\mathcal{N}}^+(\Omega_k)| &= |d_{\mathcal{N}}^+(t) - d_{\mathcal{N}}^*(t)| \\ &\le |d_{\mathcal{N}}^+(t)| + |d_{\mathcal{N}}^*(t)|. \end{aligned} \tag{26}$$

By integrating in the time interval Ω_k we obtain

$$\begin{aligned} |\Delta C_{\mathcal{N}}^+(\Omega_k)| &= \frac{1}{T} \int_{\Omega_k} |d_{\mathcal{N}}^+(t) - d_{\mathcal{N}}^*(t)| dt \\ &\le m_k^+ + m_k^* \end{aligned} \tag{27}$$

where

$$m_k^+ = \frac{1}{T} \int_{\Omega_k} |d_{\mathcal{N}}^+(t)| dt, \quad m_k^* = \frac{1}{T} \int_{\Omega_k} |d_{\mathcal{N}}^*(t)| dt, \tag{28}$$

and considering the n time intervals $\{\Omega_k : k = 1, \dots, n\}$,

$$\sum_{k=1}^{n} |\Delta C_{\mathcal{N}}^{+}(\Omega_k)| = \sum_{k=1}^{n} \frac{1}{T} \int_{\Omega_k} |d_{\mathcal{N}}^{+}(t) - d_{\mathcal{N}}^{*}(t)| dt \leq \sum_{k=1}^{n} m_k^{+} + \sum_{k=1}^{n} m_k^{*}. \quad (29)$$

The right hand side of (29) is an upper bound of the cause of extra profit. Besides, if $\sum_{k=1}^{n} m_k^{*} = 0$, then $\sum_{k=1}^{n} |\Delta C_{\mathcal{N}}^{+}(\Omega_k)| = \sum_{k=1}^{n} m_k^{+}$. Moreover, the contract $C_{\mathcal{N}}^{*}(\Omega_k)$ is maximizing the expected profit of the grand coalition, so for a large number of subintervals $\{\Omega_k : k = 1, \dots, n\}$,

$$\sum_{k=1}^{n} m_k^{*} \ll \sum_{k=1}^{n} m_k^{+}, \quad (30)$$

and we propose to use $\sum_{k=1}^{n} m_k^{+}$ as an approximation of the cause of extra profit. Larger the value of n, more accurate is the approximation.

Now, we split the approximation of the cause of extra profit among the REPs. We consider a non-negative function ϕ such that $\phi(d_i(t) d_{\mathcal{N}}^{+}(t))$ evaluates the contribution of REP i to the absolute value of the instantaneous deviation $d_{\mathcal{N}}^{+}(t)$, then

$$\sum_{k=1}^{n} m_k^{+} = \sum_{i \in \mathcal{N}} \sum_{k=1}^{n} \frac{1}{T} \int_{\Omega_k} \frac{\phi(d_i(t) d_{\mathcal{N}}^{+}(t))}{\sum_{i \in \mathcal{N}} \phi(d_i(t) d_{\mathcal{N}}^{+}(t))} |d_{\mathcal{N}}(t)| dt. \quad (31)$$

For each REP i, we define D_i as follows:

$$D_i = \sum_{k=1}^{n} \frac{1}{T} \int_{\Omega_k} \frac{\phi(d_i(t) d_{\mathcal{N}}^{+}(t))}{\sum_{i \in \mathcal{N}} \phi(d_i(t) d_{\mathcal{N}}^{+}(t))} |d_{\mathcal{N}}(t)| dt. \quad (32)$$

D_i is the the contribution of REP i to the cause of extra profit for the long term time interval $\cup_{k=1}^{n} \Omega_k$ and $D_{\mathcal{N}} = \sum_{i \in \mathcal{N}} D_i$.

Note that every REP contributes to the extra profit generation, and it should be distributed suitably to them. We propose that the aggregator allocates the extra profit proportionally to the contribution of each REP to its cause as follows.

Allocation 2 (Long-Term Profit Allocation): An allocation for the extra profit $\Delta\Pi$ accumulated during the long-term time interval $\cup_{k=1}^{n} \Omega_k$ for the grand coalition $\mathcal{N}$ that bids the optimal contracts $\{C_{\mathcal{N}}^{*}(\Omega_k) : k = 1, \dots, n\}$ but shares the short-term profit using Allocation 1 for the additive individual contract $C_{\mathcal{N}}^{+} = \sum_{i \in \mathcal{N}} C_i^{+}$ is given by:

$$\rho_i = \Delta\Pi \frac{D_i}{D_{\mathcal{N}}}, \; i \in \mathcal{N}, \quad (33)$$

where D_i is defined in (32) and $D_{\mathcal{N}} = \sum_{i \in \mathcal{N}} D_i$.

The sharing rule defined in (33) is in fact a class of profit allocations. The choice of function ϕ defines an instance of the class. Two relevant functions are the absolute value and the positive part. The absolute value $\phi(x) = |x|$ corresponds to the case where all the REPs contribute to the profit cause proportionally to the magnitude of their deviations but independently of the sign. On the other hand, the positive part $\phi(x) = (x)_+ = \max\{0, x\}$ corresponds to the case where the only the REPs whose deviation has the same sign of the aggregated deviation $d_{\mathcal{N}}^{+}$ are considered to contribute to the profit cause proportionally to the magnitude of their deviation.

In the next subsection, we shall analyze some additional properties of this class of profit allocation.

C. Extra Profit Allocation based on Cost Causation

We defined an axiomatic framework to analyze cost causation based allocations in [9]. A cost causation based allocation follows the axioms of equity, monotonicity, individual rationality, budget balance, penalty for causing cost, reward for mitigating cost.

We have shown that assuming the joint deviation with respect to the optimizing contract of the grand coalition is small, the cause of the additional profit for the time interval $\cup_{k=1}^{n} \Omega_k$ is the sum of the time average individual deviation magnitudes given by (32). Thus, we define the axioms that characterize a cost causation based allocation of the additional profit $\Delta\Pi$, using the sum of the individual time average deviation magnitudes $\{D_i : i \in \mathcal{N}\}$ as the cause of additional profit.

The following four axioms establish fairness of the allocation mechanism for the extra profit $\Delta\Pi$:

Axiom 1 (Equity): If two REPs i and j have the same contributions to the extra profit then the allocated profits must be the same, *i.e.* if $D_i = D_j$ then $\rho_i = \rho_j$.

Axiom 2 (Monotonicity): The extra profit allocation is monotonically increasing with the contribution to the extra profit, *i.e.* for any two REPs $i, j \in \mathcal{N}$, $D_i \leq D_j$ implies $\rho_i \leq \rho_j$.

Axiom 3 (Individual Rationality): The allocation $\{\rho_i : i \in \mathcal{N}\}$ is individually rational for the n time subintervals $\{\Omega_k : k = 1, \dots, n\}$ if $\rho_i \geq 0$.

Axiom 4 (Budget Balance): The allocation $\{\rho_i : i \in \mathcal{N}\}$ is budget balanced if $\sum_{i \in \mathcal{N}} \rho_i = \Delta\Pi$.

In addition, Cost Causation Principle [15] is based on the two conditions:

- Those individuals who cause costs to the system should pay for those costs.
- Those individuals who mitigate costs to the system should either incur a lower cost or be paid for helpful actions.

In our case, all the REPs are contributing to the contract deviation, so none of them must be penalized if there is an extra profit, *i.e.* $\Delta\Pi > 0$, and none of them must be rewarded if there is an extra cost *i.e.* $\Delta\Pi < 0$. The cost causation axioms for the extra profit are as follows:

Axiom 5 (Penalty for causing cost): Let the extra profit be $\Delta\Pi \leq 0$, then the allocation $\{\rho_i : i \in \mathcal{N}\}$ should be such that $\rho_i \leq 0$ for any $i \in \mathcal{N}$.

Axiom 6 (Reward for cost mitigation): Let the extra profit be $\Delta\Pi \geq 0$, then the allocation $\{\rho_i : i \in \mathcal{N}\}$ should be such that $\rho_i \geq 0$ for any $i \in \mathcal{N}$.

In the following theorem we prove that the extra profit allocation given by (33) is a cost causation based allocation if the long-term extra profit ΔP_i is positive.

Theorem 6: The extra profit allocation defined by (33) is a cost causation based allocation satisfying equity, monotonicity, and budget balance axioms. Moreover if $\Delta\Pi \geq 0$, then it also satisfies individual rationality axiom.

Proof: The extra profit allocation is cost causation based because all REPs contribute to the the contract deviation formation and therefore, all of them are rewarded if the extra profit is positive while all of them are penalized when the extra profit is negative. Equity, monotonicity and budget balance axioms hold because the extra profit allocation is proportional to the sum of time average deviation magnitudes $\{D_i : i \in \mathcal{N}\}$. Finally, $\rho_i \geq 0$ if and only if the extra profit $\Delta\Pi \geq 0$, consequently individual rationality is obtained only if the extra profit is nonnegative. □

This allocation would be satisfactory for every REPs only if it is also individually rational, *i.e.* if $\rho_i \geq 0$ for all $i \in \mathcal{N}$. Thus, the aggregator should choose $n = 24d$ sufficiently large such that the extra profit $\Delta\Pi = \sum_{k=1}^{n} \Pi_k \geq 0$. Note that since Π_k is a random variable with non-negative expected value, and $\mathbb{E}[\Delta\Pi] = \sum_{k=1}^{n} \mathbb{E}[\Delta\Pi_k] > 0$, then

$$\lim_{n\to\infty} \mathbb{P}\left\{\sum_{k=1}^{n} \Delta\Pi_k > 0\right\} = 1, \tag{34}$$

and the realized extra profit $\Delta\Pi$ attains a positive value with large probability for a sufficiently large number n of time subintervals.

D. Time interval length and maximum loss

The aggregator chooses two parameters to get a satisfactory long-term allocation of the expected profit, namely the time interval length and the maximum loss. The time interval length is the number n of short-term intervals $\{\Omega_k : k = 1, \ldots, n\}$, such that the probability that the extra profit is very small. The number n can be selected by fixing a small number $\epsilon_1 > 0$ such that

$$n = \min_{1\leq \ell}\left\{\ell : \mathbb{P}\left\{\sum_{k=1}^{\ell} \Delta\Pi_k \leq 0\right\} \leq \epsilon_1\right\}. \tag{35}$$

An important issue of the two step allocation strategy proposed here is that the aggregator needs cash for short-term losses. The short-term allocation is performed using the 'virtual' realized profit that the coalition would have been obtained by bidding the sum of contracts maximizing individual profit, but the actual realized profit is obtained using the contract maximizing the grand coalition expected profit. On average, this actual realized profit is greater than the allocated profit, however in some time intervals this allocated profit can be less than the realized profit. The extra profit is allocated after a sufficiently large number of time subintervals to ensure that it is positive. However, if the extra-profit is negative during a number of time intervals, the aggregator would lose money and needs a reserve of cash to deal with this issue.

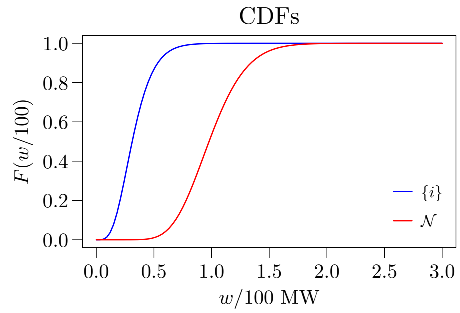

Fig. 1. Probability distribution functions of wind power of the producers

The maximum loss that the aggregator incurs by using the two-step allocation strategy during the long-term interval $\cup_{k=1}^{n} \Omega_k$ is defined as follows:

$$\mathcal{L}(\cup_{k=1}^{n} \Omega_k) = \max_{1\leq \ell\leq n}\left\{\left(-\sum_{k=1}^{\ell} \Delta\Pi_k\right)_+\right\} \tag{36}$$

The maximum loss is a random variable, so it takes different values depending on the realized power productions. However, we can still assign probabilities to different levels of maximum loss. This way, by fixing a small number $\epsilon_2 > 0$ we can say that the maximum loss is greater than L with a probability ϵ_2 for

$$L = \min_{Q}\left\{Q : \mathbb{P}\left\{\mathcal{L}(\cup_{k=1}^{n} \Omega_k) \geq Q\right\} \leq \epsilon_2\right\} \tag{37}$$

Explicit expressions for the long-term interval length n and the maximum loss L cannot be easily obtained, however if the probability distribution functions of the wind power production are known or can be estimated from empirical data, then they can be computed by statistical inference [16], [17]. In order to operate, the aggregator needs an initial amount of cash greater than the maximum loss L to guarantee that the REPs are receiving their payments for the time subintervals. At the end of the long-term time interval, this amount is recovered for the new long-term time interval.

V. Numerical Example

In this Section, we develop a numerical example in order to explain the developed theory. Here we consider three wind power producers that are geographically distant such that probability distribution of their wind production is independent. We assume that they have the same nameplate capacity $W_i = 100$ MW, and the wind power production (normalized by the nameplate capacity) is constant during the time interval Ω_k and is modeled by a Gamma random variable, *i.e.* $w_i/W_i \sim \Gamma(5, \frac{1}{15})$. The cumulative probability distributions (CDF) of each wind farm and that of the grand coalition are depicted in Figure 1. The gamma distribution has been frequently used in the literature to model wind speed [18].

For simplicity of computation, we assumed that the market clearing price is constant throughout the long-term interval. The market clearing price is $p = \$20$, the penalty prices for shortfall and excess (in US dollars) are modeled as statistically independent uniformly distributed random variables $q \sim \mathcal{U}_{[40,100]}$ and $\lambda \sim \mathcal{U}_{[0,40]}$.

We have drawn a set of 8760 samples of the random variables $\{(w_1, w_2, w_3, q, \lambda)_k : k = 1, \dots, 8760\}$ to study a complete year of operation. Here we assume that each REP will maximize her expected profit to derive her contract at each interval Ω_k, *i.e.* $C_i^+(\Omega_k) = C_i^*(\Omega_k) = F_i^{-1}(\gamma_k; \Omega_k)$, where $F_i^{-1}(\gamma_k; \Omega_k)$ denotes the quantile function of the wind power for the time interval Ω_k. From these data we have obtained the contracts $\{C_i^* : i \in \mathcal{N}\}$ and $C_\mathcal{N}$ maximizing the expected profit of each REP and the expected profit of the grand coalition, respectively, for each hour $\{\Omega_k : k = 1, \dots, 8760\}$. The deviations are obtained as $\{d_i = w_i - C_i^* : i \in \mathcal{N}\}$ and $d_\mathcal{N} = w_\mathcal{N} - C_\mathcal{N}^*$, also for each hour of the year. Assuming that the aggregator bids the contract maximizing the expected profit $C_\mathcal{N}^*$, but the REPs expect to be paid according to their individual optimal contracts C_i^*, we can compute the short-term stabilizing profit allocation (19) and the extra profit $\Delta\Pi_k$ (22) for each hour $\{\Omega_k : k = 1, \dots, 8760\}$.

The aggregator needs to compute the length of the long-term time interval for allocating the extra profit and the maximum loss that could be incurred.

We have drawn data from the known probability distributions of the wind production and prices and we have computed extra profit data $\{\Delta\Pi_k : k = 1, \dots, M\}$ for $M = 720000$ different hours to estimate the number n of short-term time intervals and the maximum loss L. In Figure 2 we represent the estimated probability of negative long-term extra profit

$$\mathbb{P}\left\{\sum_{k=1}^{24n} \Delta\Pi_k \leq 0\right\} \tag{38}$$

as a function of the number of days n. We can see that the probability is monotonically decreasing with the length of the time interval. For $n = 20$ days, this probability is less than 0.01, and for $n = 30$ days is less than 0.003. Consequently, we can infer from the set of data that the extra profit $\Delta\Pi$ is positive for a long-term interval of a month with a 99.7% of probability.

In Figure 3, we represent the estimated probability of the maximum loss of the aggregator. This probability is given by

$$\mathbb{P}\left\{\max_{1\leq \ell \leq n}\left(-\sum_{k=1}^{24\ell} \Delta\Pi_k\right)_+ \geq L\right\} \tag{39}$$

and is monotonically decreasing with the loss L. For $L = 5000$, the probability is around 0.02, and for $L = 10000$ the probability is less than 0.002, so we infer from the data set that the aggregator can cover any potential loss during a month with a 99.8% of probability by making a reserve of \$10000.

In Table I we show the result of the short term allocation. For each hour, the aggregator computes the profit to be allocated as if the coalition would bid the sum of the

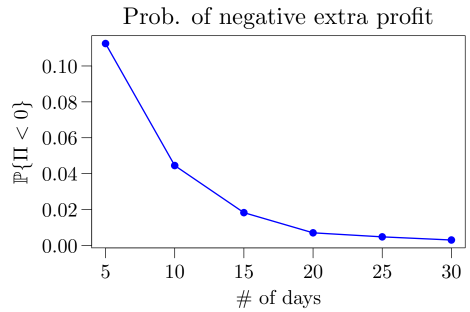

Fig. 2. Probability of negative extra profit as a function of the number of days

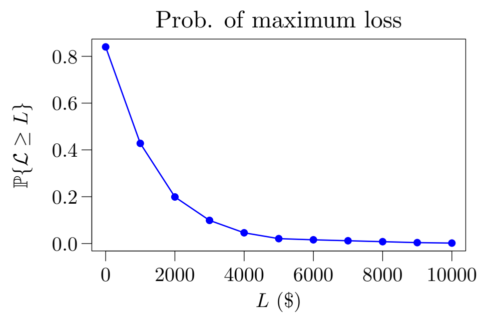

Fig. 3. Probability of aggregator loss as a function of the loss

individual optimal contracts, $C_\mathcal{N}^+$ and allocates this profit using (19). The extra profit is computed as the difference between the actual realized profit (by bidding $C_\mathcal{N}^*$) and the allocated profit. The results in Table I are presented by natural months, adding every hour of each day of the month. Note that the extra profit is always positive for each month, as it was anticipated by the statistical analysis of the data set.

The monthly extra profit is distributed using the allocation rule (33) where D_i is given by (32) and $D_\mathcal{N} = \sum_{i\in\mathcal{N}} D_i$. We have considered two cases. In the first case, the function that evaluates the short-term contribution to the contract deviation is the positive part, *i.e.* $\phi(x) = (x)_+$, and the extra profit allocated to each REP is given in Table II. In the second case, the absolute value, *i.e.* $\phi(x) = |x|$, was used

TABLE I
SHORT-TERM ALLOCATION BY MONTH (IN \$)

Month	Π	ξ_1	ξ_2	ξ_3
1	806535.03	263629.70	262820.77	257701.72
2	754921.72	230388.88	256775.94	238833.31
3	826081.67	245650.90	261214.45	291464.22
4	832788.99	266696.63	247766.43	280206.37
5	858124.58	267995.80	268892.57	279127.28
6	794049.79	264643.07	247653.63	253739.15
7	756480.93	266918.14	238464.74	239571.89
8	860153.00	264740.79	272370.23	282627.69
9	756958.64	249152.70	240407.31	249603.51
10	850838.04	263717.51	295927.52	257384.71
11	773244.53	233985.11	251356.44	264959.97
12	824695.68	256979.71	284642.61	250715.49

TABLE II

EXTRA PROFIT ALLOCATION BY MONTH $\phi(x) = x_+$ (IN $)

Month	$\Delta\Pi$	ρ_1	ρ_2	ρ_3
1	22382.84	7689.31	7186.28	7507.24
2	28923.59	10211.38	8751.15	9961.06
3	27752.10	9427.88	9672.47	8651.75
4	38119.56	12916.69	13589.22	11613.66
5	42108.93	13439.45	14993.39	13676.09
6	28013.95	9506.59	8710.70	9796.66
7	11526.17	3974.34	3887.26	3664.57
8	40414.29	14251.53	13208.26	12954.50
9	17795.13	5860.19	6468.04	5466.90
10	33808.31	11105.16	10652.52	12050.63
11	22943.01	7486.49	8066.27	7390.25
12	32357.87	10716.49	11077.25	10564.14

TABLE III

EXTRA PROFIT ALLOCATION BY MONTH $\phi(x) = |x|$ (IN $)

Month	$\Delta\Pi$	ρ_1	ρ_2	ρ_3
1	22382.84	7698.23	7187.39	7497.22
2	28923.59	10143.64	8924.00	9855.95
3	27752.10	9408.88	9571.45	8771.78
4	38119.56	12911.79	13358.39	11849.38
5	42108.93	13511.44	14835.04	13762.45
6	28013.95	9544.25	8836.38	9633.32
7	11526.17	4025.65	3845.85	3654.66
8	40414.29	14210.40	13293.17	12910.72
9	17795.13	5879.49	6368.87	5546.77
10	33808.31	11239.26	10751.14	11817.91
11	22943.01	7569.13	7915.86	7458.03
12	32357.87	10728.79	11173.91	10455.18

and the results are presented in Table III. In this last case, the short-term individual deviations having opposite sign to the contract deviation $\Delta C_{\mathcal{N}}^{+}$ are considered that do not contribute to the extra profit. Consequently, REPs having less deviations in the opposite direction to the contract deviation, which is the cause of extra profit, get more profit allocated.

We conclude the study showing the maximum loss of the aggregator for each month in Table IV. Note that the maximum loss is not higher than $6000 for any month. This result is also in accordance with the statistical study shown in Figure 2.

VI. CONCLUSION

In this paper, we developed a two step strategy to improve the long-term benefit of a coalition of renewable energy producers who are interested to have stable allocation of their short term revenues. This strategy attains a twofold aim, first it promotes aggregation by using a stabilizing allocation of the short-term realized profit, and second, it allows the REPs to maximize the expected profit of the grand coalition. As penetration of renewable energy becomes deeper, there will be greater pressure on the producers to operate under market mechanisms. Under such conditions, ideas developed in this paper will prove valuable to take advantage of geographic diversity. It will be necessary to further develop these ideas to take into account network constraints and locational marginal prices, a worthy topic for future research.

TABLE IV

AGGREGATOR'S MAXIMUM LOSSES BY MONTH (IN $)

Month	Loss	Month	Loss
1	1985.45	7	554.10
2	797.33	8	1599.67
3	5018.52	9	1419.48
4	1304.61	10	132.88
5	0.00	11	1046.92
6	775.87	12	0.00

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